MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

THIRD/fourth YEAR …… SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONIC ENGINEERING)

BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION TECHNOLOGY)

EEE 415/ SPH 315: ELECTRODYNAMICS AND INSULATING MATERIALS

DATE: **TIME:**

INSTRUCTIONS

- *This paper consists of **Five** Questions*
- ***Question 1** is **Compulsory**.*
- *Attempt **Any Other Two Questions***

Important Physical Constants

Conductivity of copper $\sigma = 5.8x10^7 S/m$

$\mu_0 = 4\pi x10^{-7} H/m$

$\epsilon_0 = 8.854x10^{-12} F/m$

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Beginning from Ampere's Law, show that (2 marks)

$$\vec{\nabla} x \vec{\mathrm{H}} = \frac{d\vec{\mathrm{D}}}{dt} + \vec{\mathrm{J}}$$

b) Beginning from Faraday's Law, show that (2marks)

$$\vec{\nabla} x \vec{\mathrm{E}} = -\frac{d\vec{\mathrm{B}}}{dt}$$

c) Applying a) and b), show that (5 marks)

$$\gamma = [j\omega\mu(\sigma + j\omega\epsilon)]^{1/2}$$

All the symbols carry their usual meaning

d) For complex μ and ϵ in c) above, show that for most materials (12 marks)

$$\alpha = \omega\sqrt{\frac{\mu\epsilon'}{2}}\left[\sqrt{1 + \ (Tan\ \delta)^2} - 1\right]^{1/2}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon'}{2}}\left[\sqrt{1 + \ (Tan\ \delta)^2} + 1\right]^{1/2}$$

All symbols carry their usual meaning

e) Using c) above, obtain the expression of the phase velocity for a low loss dielectric (5 marks)

f) Using c) above, derive the expression of the *skin depth* for a good conductor (4 marks)

QUESTION TWO (20 MARKS)

Given that an electromagnetic system is defined by the relation

$$\nabla.\left(\vec{E} x \vec{H}\right) = \overrightarrow{H^*}\left(\vec{\nabla} x \vec{\mathrm{E}}\right) - \overrightarrow{E^*}(\vec{\nabla} x \overrightarrow{\mathrm{H}})$$

All the symbols carry their usual meaning.

a) Define $\overrightarrow{S_P}\ and\ \eta$ in terms of $\vec{E}\ and\ \vec{H}$ (4 marks)

b) Analyze how the power delivered by a source J_s to the system is utilized (7 marks)

c) Derive the $\overrightarrow{S_{av}}$ for elliptical polarization in terms of $\vec{E}\ and\ \vec{H}$ (6 marks)

d) Given that

$$\vec{E} = 10\,\mathrm{Cos}\left(\omega t + \frac{4\pi}{3} z\right) U_x V/m$$

And that

$$\vec{H} = \frac{U_z x \vec{E}}{\mu_0} \ A/m$$

Determine the *average power per unit area* transported by the wave at a frequency of 500MHz from *First Principles* (3 marks)

QUESTION THREE (20 MARKS)

a) State *Helmholtz Theorem* (2 marks)

b) The Magnetic and Electric Fields in a medium can be defined by

$$\vec{B} = \vec{\nabla}\, x\vec{A} + \nabla\phi$$

$$\vec{E} = \vec{\nabla}\, x\vec{F} + \nabla\phi$$

where $\vec{A}$ and $\vec{F}$ are vector potential functions and ϕ is a scalar potential function ϕ. Show that (6 marks)

$$\nabla^2\phi = -\frac{\rho}{\epsilon}$$

$$\nabla^2\vec{A} = -\mu\vec{\mathrm{J}}$$

c) Given that

$$\vec{\nabla} x \vec{\mathrm{H}} = \frac{d\vec{\mathrm{D}}}{dt} + \vec{\mathrm{J}}$$

$$\vec{\nabla} x \vec{\mathrm{E}} = -\frac{d\vec{\mathrm{B}}}{dt}$$

Show that the *Homogeneous Helmholtz Equation (HHE)* is given by

$$\nabla^2\vec{\mathrm{E}} + \mathrm{K}^2\vec{\mathrm{E}} = 0$$

d) Show that the *Inhomogeneous Helmholtz Equation (IHE)* is given by

$$\nabla^2\phi + K^2\phi = -\frac{\rho}{\epsilon}$$

$$\nabla^2\vec{A} + K^2\vec{A} = -\mu\vec{\mathrm{J}}$$

QUESTION FOUR (20 MARKS)

a) Derive the *Reduced Maxwell's Equations* (RMEs) for a guided wave (6 marks)

b) Write the expressions in a) above for a TEM waves (2 marks)

c) For the TEM waves propagating in a Lossless Medium in +Z direction, derive the following

i) K_0 (2 marks)

ii) η_0 (2 marks)

iii) Expressions of $\vec{E}(z,t)$ and $\vec{H}(z,t)$ (4 marks)

Explain all the symbols used

d) The Electric Field intensity of a linearly polarized uniform wave propagating in the +z direction in sea water at the surface $z = 0$ is

$$\vec{E} = \hat{x}100\cos(\pi x 10^7)V/m$$

Sea water has $\epsilon_r = 72$, $\mu_r = 1$ and $\sigma = 4S/m$.

Determine the expression for $\vec{E}(z,t)$ and $\vec{H}(z,t)$ at $z = 0.8m$ as a function of time (4 marks)

QUESTION FIVE (20 MARKS)

a) A parallel polarized wave at an oblique incidence on a dielectric interface is as shown in Figure Q 5a). Derive the expressions for the *Brewster and Transmission Angles*. All symbols carry their usual meaning (7marks)

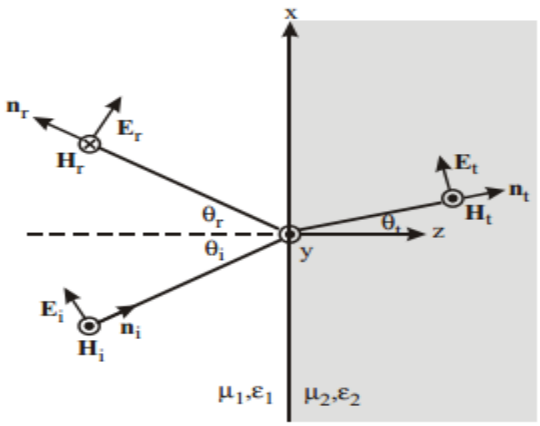

Figure Q 5a)

b) The same wave in Question 5a) is perpendicularly polarized. Derive the expressions for the Fresnel's Coefficients of Reflection and Transmission. All symbols carry their usual meaning (7 marks)

c) The dielectric constant of pure water is 80. Determine the following

(i) Brewster Angle and Angle of Transmission for Question 5a) above (3 marks)

(ii) Reflection and Transmission coefficients at $\theta_i = \theta_{Bp}$ (3 marks)