MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

SMA 332: METHODS OF APPLIED MATHEMATICS

DATE: 11/4/2023 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS: Answer Question One and any other two questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) State the equation representing each of the given canonical forms of 2nd order PDEs: hyperbolic, parabolic and elliptic forms. (3 marks)

b) Decompose the Laplace equation $u_{xx} + u_{yy} = 0$, $0 < x, y < L$ into ordinary differential equations (ODEs) in variables X and Y. (5 marks)

c) Sate the Fourier Cosine series expansion of a function $f(x)$ defined over the interval, $-L < x < L$. (2 marks)

d) Show that the Fourier coefficient a_n of the Fourier Cosine series, you have defined in b) above is given by; $a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx, \quad n = 1,2,3, \ldots$ (5 marks)

e) Given that $u = u(x,y)$ and the change of variables $\theta = \theta(x,y), \eta = \eta(x,y)$ such that $u_x = u_\theta \theta_x + u_\eta \eta_x$, show that;

$u_{xx} = u_{\theta\theta}\theta_x^2 + 2u_{\theta\eta}\theta_x\eta_x + u_{\eta\eta}\eta_x^2 + u_\theta\theta_{xx} + u_\eta\eta_{xx}$ (5 marks)

f) Use the First Shifting theorem to determine the inverse Laplace transform of the quotient (5 marks)

$$\frac{s+2}{s^2+2s+2}$$

g) Show that the Laplace transform $\frac{d^2y}{dt^2}$ is given by $s^2\bar{y}(s) - sy(0) - y'(0)$

(5 marks)

QUESTION TWO (20 MARKS)

a) Given the 2nd order partial differential equation (PDE) $2u_{xx} + 2yu_{yy} + u_y = 0$

i) Determine the regions in the xy-plane where this equation is hyperbolic, parabolic and elliptic. (3 marks)

ii) For $y < 0$ transform the given equation into a hyperbolic canonical form. (10 marks)

b) Obtain the Fourier series expansion for the function

$$f(x) = 2x, \quad -\pi < x < \pi$$ (7 marks)

QUESTION THREE (20 MARKS)

a) Use partial fractions to determine the inverse Laplace transform of the quotient:

$$\frac{6+s^3}{(s+2)s^3}$$ (10 marks)

b) Solve the following 2nd differential equation using Laplace transform method;

$y'' + 2y' + 2y = 0$ subject to $y(0) = 1,\ y'(0) = -1$ (10 marks)

QUESTION FOUR (20 MARKS)

a) A heavy-duty wire of length a that is fixed at $x = 0$ and free to move at the other end $x = a$ is initially at rest and subjected to a constant force. Assuming that the wire's displacement $u(x, t)$ happens only in the x direction and $u_x(a, t) = D$ where D is the displacement per unit length.

i) Show that the Laplace transform of the associated wave equation is given by the ODE $\frac{d^2U(x,s)}{dx^2} = \frac{s^2}{c^2}U(x, s)$ (5 marks)

ii) Show that the solution to the ODE given in 4 a) i) above is

$U(x, s) = \frac{cD\sinh(sx/c)}{s^2\cosh(sa/c)}$ (10 marks)

b) Determine the Laplace transform of $f(x) = \cos ax$. (5 marks)

QUESTION FIVE (20 MARKS)

a) A thin bar of length π units is place in boiling water (temperature 100^oC). After reaching 100^oC throughout, the bar is removed from the boiling water. With the lateral sides kept insulated, suddenly, at time $t = 0$, the ends are immersed in a medium with constant freezing temperature 0^oC. Taking c=1, find the temperature $u(x, t)$ for $t > 0$ using the separation of variables method (10 marks)

b) Obtain the general solution to the 2nd order PDE $yu_{tt} - k^2yu_{yy} = 2k^2u_y$ where k is a constant. (10 marks)