



# MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

BACHELOR OF ARTS

SMA 433: PARTIAL DIFFERENTIAL EQUATIONS II

DATE: 30/4/2019

TIME: 8:30 – 10:30 AM

## INSTRUCTIONS:

Attempt question *one (compulsory)* and any other *two questions*.

## QUESTION ONE (Compulsory) (30MARKS)

- a) Define a second order linear partial differential equation (3 marks)
- b) Define a Fourier transform of a function  $f(x)$  (4 marks)
- c) Consider the convolution theorem

$$F^{-1}[\bar{F}(\xi)\bar{G}(\xi)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(u)g(x-u)du$$

Prove the theorem. (4 marks)

- d) Consider the equation  $2u_{xx} + 3u_{xy} + u_{yy} = 0$ .
  - i) Classify the equation (4 marks)
  - ii) Determine its characteristic equation (5 marks)
  - iii) Transform the equation to canonical form (5 marks)
  - iv) Solve the equation (5 marks)

**QUESTION TWO (20 MARKS)**

An infinitely long string having one end at  $x = 0$  is initially at rest on the  $x$  – axis . At  $t = 0$  the end  $x = 0$  begins to move along the  $u$  – axis in a manner described by  $u(0,t) = a \cos \sigma t$  .

Determine the displacement  $u(x,t)$  of the string at any point at any time.

**QUESTION THREE (20MARKS)**

Solve the wave equation  $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$

Given that  $u(x,0) = f(x)$  and  $u_t(x,0) = g(x)$  .

**QUESTION FOUR (20MARKS)**

Determine the characteristic of the Tricomi equation  $y u_{xx} + u_{yy} = 0$  in the lower-half plane  $y < 0$ . Put this equation into canonical form in the upper half- plane  $y > 0$ .

**QUESTION FIVE (20 MARKS)**

Determine the Fourier transforms for the following functions;

i)  $f(x) = e^{-a|x|}$  (10 marks)

ii)  $f(x) = e^{-ax^2}$  (10 marks)