



# MACHAKOS UNIVERSITY

University Examinations for 2018/2019 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

THIRD YEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN AGRI-BUSINESS MANAGEMENT AND TRADE

EAE 414: THEORY OF FINANCE

DATE: 26/7/2019

TIME: 8:30 – 10:30 AM

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## INSTRUCTIONS:

- (i) Answer question one (Compulsory) and any other two questions
- (ii) Do not write on the question paper
- (iii) Show your workings clearly

## QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Define the following as used in finance
  - i) Law of one price (2 marks)
  - ii) Bid-Ask spread (2 marks)
  - iii) Arbitrage and strong arbitrage (2 marks)
  - iv) Risk free asset (2 marks)
  - v) Redundant Security (2 marks)
- b) Mercy, a K14 student cannot figure out why market completeness is critical in a one period framework security market. Explain to him what market completeness is as well as its implications. (7 marks)
- c) Consider a financial market with a money account, a stock, and a European call option on the stock with strike price of KES196. Suppose there are two future states. If state 1 realizes, the stock price declines to KES 168 from the current price of KES200. If state 2 happens, the stock price rises to KES224. Suppose the interest rate on the money account is 5% no matter the state and assuming that the payoffs on the stock and the money account are the stock prices and compounded money value respectively in the different date 1 states.

- i) Complete the table below for the payoffs and clearly state the rationale for your answers (2 marks)

|               | Date 0 | Date 1 (Payoffs) |         |
|---------------|--------|------------------|---------|
|               | Price  | State 1          | State 2 |
| Money Account | 1      |                  |         |
| Stock         | 200    |                  |         |
| Call Option   | $C_0$  |                  |         |

- ii) At what price should we price the call option to avoid arbitrage (3 marks)
- iii) Would there be an arbitrage if the price were 15 or 25 (8 marks)

## QUESTION TWO (20 MARKS)

A security market has three securities. Security one has a payoff of 10 in state one, -5 in state two and a payoff of 3 in state 3. Security two has a payoff of -7 in state one, 14 in state two and a payoff of -5 in state 3. Security three has payoff of -1 in state one, -5 in state two and a payoff of 7 in state 3. If the security prices are 55, 27 and 18 for security one, two and three respectively;

- i) Form the payoff matrix for this market (1 mark)
- ii) Find and interpret the state prices given that  $p_j = \sum_s x_{js} \frac{u'_1(c_0, c_1)}{u'_0(c_0, c_1)}$  (value of a security = sum [payoffs in future states X state prices]). (10 marks)
- iii) Find and interpret the risk-free rate in this market (1 mark)
- iv) Find the price of a put option placed on the 3<sup>rd</sup> security if its strike price KES 5 given that the payoffs in the payoff matrix represent the future prices of the securities in the various states in date 1 (3 marks)
- v) What is the replicating portfolio for this put option? Does the replicating portfolio have the same price as the put option given the date zero prices of security 1, 2 and 3 (5 marks)

## QUESTION THREE (20 MARKS)

- a) Given that  $y = mx + c$  where m and c are constants, use the properties of mean and variance to prove that;
- i)  $E(y) = c + mE(x)$  Where E is the expectations operator (1 mark)
- ii)  $Var(y) = m^2 Var(x)$  (2 marks)

- b) Given that securities X and Y are perfectly correlated such that

$$Y = 11.15789 - 0.315789X \text{ and the probability distribution of X is;}$$

| probability | $x_i$ % |
|-------------|---------|
| 0.2         | 11      |
| 0.2         | 9       |
| 0.2         | 25      |
| 0.2         | 7       |
| 0.2         | -2      |

Find;  $E(X)$  and  $Var(X)$  (3 marks)

$E(Y)$  and  $Var(Y)$  (3 marks)

- c) Write down the equations for expected portfolio return and variance for a portfolio consisting of security X and Y if  $a$  % is invested in security X and  $b$  % is invested in Y (2 marks)
- d) Use the equations in (c) above to complete the following table given that  $cov(x, y) = -0.0024$ .  $E(R_p)$  and  $\sigma(R)$  are expected portfolio returns and standard deviation of portfolio returns respectively. (6 marks)

| Percentage in X | percentage in Y | $E(R_p)\%$ | $\sigma(R) = \sqrt{\text{var}(R_p)}$ | $\sigma\%$ |
|-----------------|-----------------|------------|--------------------------------------|------------|
| 100             | 0               |            |                                      |            |
| 75              | 25              |            |                                      |            |
| 50              | 50              |            |                                      |            |
| 25              | 75              |            |                                      |            |
| 0               | 100             |            |                                      |            |

- e) Use the table in (d) above to demonstrate the significance of portfolio diversification (3 marks)

#### QUESTION FOUR (20 MARKS)

- a) Explain four financial problems faced by individuals and corporations. (8 marks)
- b) State the agents consumption portfolio choice problem under short sale restrictions (2 marks)
- c) Let there be two securities: a risky security with return denoted by  $r$  and a risk free security with return  $r^*$ . For a portfolio  $(h_1, h_2)$  such that  $p_1h_1 + p_2h_2 = w$ , let  $a = p_2h_2$  denote the amount invested in the risky security. Required;
- Represent portfolio  $(h_1, h_2)$  in terms of wealth (3 marks)
  - What significance does the reformulation in (i) above have? (2 marks)
  - State the payoff of the portfolio in (i) above in terms of  $w$ ,  $a$ ,  $r$ , and  $r^*$  (2 marks)

- iv) State the agents portfolio choice problem given that the representative agents utility function is  $u = V(z)$ , where  $z$  is a portfolio payoff. (3 marks)

**QUESTION FIVE (20 MARKS)**

- a) To what extent are the assumptions on characterization of securities and portfolios applicable to security markets in Kenya (10 marks)
- b) In a one-period two-state economy the risk-free interest rate over the period is 25%. An asset that pays out 100 in state 1 and 200 in state 2 trades at a price of 110. What is the no-arbitrage price of a second risky asset that pays out 200 in state 1 and 100 in state 2? (6 marks)
- c) Distinguish between limited and unlimited arbitrage. (4 marks)