MACHAKOS UNIVERSITY

University Supplementary Examinations

BACHELOR OF SCIENCE MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF EDUCATION, BACHELOR OF MATHEMATICS

SMA 431. DIFFERENTIAL GEOMETRY

INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a. Two vectors $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A} = 6\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}$

(i) Compute the scalar and vector products of the two vectors (5 Marks)

(ii) Find the angle between $\mathbf{A}$ and $\mathbf{B}$. (2 Marks)

(iii) Determine a unit vector perpendicular to the plane of $\mathbf{A}$ and $\mathbf{B}$. (3 Marks)

b. A space curve is defined by $\mathbf{r} = 2\mathbf{i}\sin 3t + 2\mathbf{j}\cos 3t + 8\mathbf{k}t$.

Determine velocity and acceleration of a particle moving along the curve. (2 marks)

Find the magnitude of the velocity and acceleration. (2 marks)

c. A curve is defined by the vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + 2\mathbf{k}$ where $x = 1 + t^2$, $y = 4t - 3$ and $z = 2t^2 - 6t$.

i) Determine the tangent and unit tangent to the curve. (3 marks)

ii) Determine the curvature and radius of curvature at any point on the curve. (4 marks)

d. Determine the length of the arc of the curve $\mathbf{r} = e^t \cos t\mathbf{i} + e^t \sin t\mathbf{j} + e^t\mathbf{k}$ from $t = 0$ and $t = \pi$. (5 marks)

e. Find the curvature and torsion for the space curve $x = t$, $y = t^2$ and $z = t^3$ called the twisted cubic. (4 marks)

QUESTION TWO (20 MARKS)

a. List any three types of planes of a space curve giving the equation for each. (6 marks)

b. Find a unit normal to the surface $\phi(x, y, z) = xy - z^2 = 0$ at the point $P(1,4,-2)$. (3 marks)

c. Find the equation of the circle of curvature of $2xy + x + y = 4$ at the point $(1,1)$. (11 marks)

QUESTION THREE (20 MARKS)

a. Consider the surface $\mathbf{r} = \mathbf{i}uv + \mathbf{j}(uv - v) + \mathbf{k}(uv - u)$. Calculate the first quadratic differential form in general and also when $u = -1$ and $v = 1$ (5 marks)

b. Given that $f(x) = \sqrt{1-x}$ on $\left[0, \frac{1}{2}\right]$, find the surface area of the curve rotated on the x axis (7 marks)

c. Find the $\frac{dS}{dx}$ and $\frac{dS}{dy}$ on the ellipse $x^2 + 4y^2 = 8$ (8 marks)

QUESTION FOUR (20 MARKS)

a. Find the fundamental magnitudes of the first and the second order of the surface; $\mathbf{r} = \mathbf{i}u + 2\mathbf{j}v + 2\mathbf{k}uv$ (10 marks)

b. Find the curvature of the parabola $y^2 = 12x$ at the points $(3,6)$ and $(0,0)$ (10 marks)

QUESTION FIVE (20 MARKS)

a. Compute the normal and the geodesic curvature of the circle $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \mathbf{k}$ on the elliptic parabolic $\mathbf{r}(u,v) = u\mathbf{i} + v\mathbf{j} + (u^2 + v^2)\mathbf{k}$ (10 marks)

b. Compute the principal curvatures and principal directions for a cylinder with the z axis as its axis and circular cross section of unit radius whose parametrization is given as $\mathbf{r}(\mathbf{u}, \mathbf{v}) = (\cos v, \sin v, u)$ (10 marks)