# Machakos University College

ISO 9001:2008 Certified

(A Constituent College of Kenyatta University)

***School of Engineering Supplementary Examinations for the second year***

***Second semester Degree students August 2016***

***Engineering Mathematics VIII, ECU 203 Supplementary Examinations August 2016***

Engineering mathematics VIII

Attempt **Question one (compulsory)** and any ***other two questions***.

1. a.) Define the following terms as used in differential equations context ;
   - i.) Analytic function (2mks)
   - ii.) Power series solution (2mks)
   - iii.) Ordinary point of a differential equation (2mks)
   - iv.) Regular singular point of a differential equation (2mks)

   b.) using the power series method solve the differential equation $y' - xy = 0$ composing your result to those obtained analytically. (5mks)

   c.) determine the polynomial solution to the Legendre equation

   $$(1-x^2)y'' - 2xy' + 12y = 0$$ (5mks)

   d.) using the Frobenius method solve the differential equation

   $$4xy'' - 2y' + y = 0$$ (5mks)

   e.) determine the Laplace transform of $f(t) = e^{at}$ where $a$ is a constant. (3mks)

   f.) determine the half range sine Fourier series representing the function $f(x)$ defined by;

   $$f(x) = 1 + x \quad ; \quad 0 < x < \pi$$

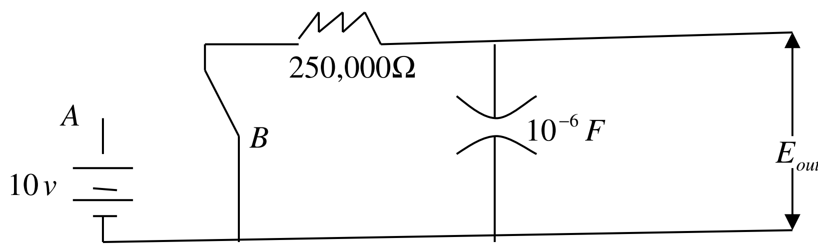
   $$f(x) = f(x + 2\pi)$$ (4mks)

2. a.) use Laplace transforms method to solve the second order differential equation

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 2e^{3t} \text{ given that at } t = 0 \; ; \; x = 5 \quad and \quad \frac{dx}{dt} = 7$$ (7mks)

b.) determine the inverse of $\dfrac{5s+1}{(s-4)(s+3)}$ (3mks)

c.) suppose the capacitor in the circuit whose diagram is given below initially has zero charge and that there is no initial current. At time $t = 2 \sec onds$ , the switch is thrown from position B to A, held there for $1 \sec ond$ , then switched back to B. calculate the output voltage $E_{out}$ on the capacitor. (10mks)




3. a.) use the Bessel's function identities to express $J_3(x)$ in terms of $J_0(x)$ and $J_1(x)$

(3mks)

b.) Evaluate $\int x^3 J_0 dx$ (3mks)

c.) determine $J_{\pm\frac{3}{2}}$ in terms of the elementary functions $\sin x$ and $\cos x$ (4mks)

d.) show that $xF(1,1,2;-x) = \ln(1+x)$ (4mks)

e.) determine the general solution to Bessel's equation of order zero. (6mks)

4. a.) determine the Fourier's series expansion for;

$$f(x) = \begin{cases} 0 & ; -\pi \le x < \frac{-\pi}{2} \\ \cos x & ; \frac{-\pi}{2} \le x < \frac{\pi}{2} \\ 0 & ; \frac{\pi}{2} \le x < \pi \end{cases}$$ (4mks)

b.) determine a Fourier cosine series for $f(x) = e^x$ on $(0, \pi)$ (4mks) c.) i.) define a Dirac delta function (1mk)

ii.) state the convolution theorem of Fourier transforms. (3mks)

iii.) given the functions $f(t)$ and $g(t)$ and their convolution is;

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(x)g(t-x)dx = h(t)$$

Where $*$ denotes the convolution operation.

If $f(t) = u(t)$ and $g(t) = \begin{cases} \sec^2 t & |t| < \dfrac{\pi}{4} \\ 0 & otherwise \end{cases}$

Where $u(t)$ is the Heaviside function.

Show that $u(t) = \dfrac{1+\tan^2 t}{1+\tan t}$ (4mks)

d.) find the inverse transform of $f(\omega) = \dfrac{1}{2\pi(a+j\omega)^2}$ where $a>0$ (5mks)

5. a.) Solve $\dfrac{\partial^2 u}{\partial x \partial y} = \sin x \sin y$ subject to the conditions;

$$\frac{\partial u}{\partial x}\left(x, \frac{\pi}{2}\right) = 2x$$

$$u\ (\pi, y) = 2\sin y$$

(5mks)

b.) use the method of separation of variables to find the solution of the equation;

$\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{k}\dfrac{\partial u}{\partial t}$ which satisfies the conditions;

i.) $u(0,t) = u(a,t) = 0 \qquad \forall\ t \geq 0$

ii.) $u(x,0) = x$ $\quad$ $for\ 0 \le x \le a$ (15mks)