MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

THIRD YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN
BACHELOR OF SCIENCE IN MATHEMATICS
BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE
BACHELOR OF EDUCATION (SCIENCE)

SMA 302: GROUP THEORY

DATE:23RD JANUARY, 2025 **TIME: 8.30 AM – 10.30 AM**

INSTRUCTIONS:
Answer question ***ONE (Compulsory)*** *and* ***any other TWO*** *questions*

QUESTION ONE *(COMPULSORY)* (30 MARKS)

a) Define * on $\mathbb{R}^+$ by $a*b=ab/2$. Show that $\mathbb{R}^+$ with the operation * is a group. (5 marks)

b) Prove that the identity element of a group is unique. (5 marks)

c) Let $G=Z_6=\{0,1,2,3,4,5\}$, the group of integers modulo 6 under addition. Determine the order of 0, 1, 2,3,4,5 (5 marks)

d) Prove that every cyclic group is abelian. (4marks)

e) Let $G=\{\pm 1,\pm i,\pm j,\pm k\}$, the quarternion group. Determine all the left cosets of $H=\{1,-1\}$ in G. (4 marks)

f) If $f: G \to G'$ is an isomorphism of G with G' and e is the identity of G then prove that $f(e)$ is the Identity in G'. (4 marks)

g) What is the order of sylow 3-subgroups of a group of order 54? (3 marks)

QUESTION TWO (20 MARKS)

a) List all the element of S_4 (5 marks)

b) State and proof Lagrange's theorem. (8 marks)

c) If A is a non empty set and S_A a collection permutations of A. Then prove that S_A is a group under permutation multiplication. (7 marks)

QUESTION THREE (20 MARKS)

a) Let S be the set of all real numbers except -1. Define * on S by $a*b=a+b+ab$ .Show that $(S,*)$ is a group (5 marks)

b) Prove that every group of prime order is cyclic (5 marks)

c) Prove that the collection of all even permutations of a finite set of n elements form a subgroup of order $\frac{n!}{2}$ (5 marks)

d) Prove that if G is a group and $a \in G$, then $H = \{a^n | n \in Z\}$ is a subgroup of G and is the smallest subgroup which contains a. (5 marks)

QUESTION FOUR (20 MARKS)

a) Prove that the order of an element of a finite group divides the order of the group. (5 marks)

b) Prove that every subgroup of an abelian group is a normal subgroup. (5 marks)

c) Prove that if G is a group with binary operation * then the right and the left cancellation holds in G. (5 marks)

d) Prove that the centre of a group is a normal subgroup of the group. (5 marks)

QUESTION FIVE (20 MARKS)

a) Prove that a subgroup of a cyclic group is cyclic. (5 marks)

b) Let $G=S_3$ and consider the subgroup $H=\langle(13)\rangle=\{1,(13)\}$, determine all the conjugates of H (5 marks)

c) Prove that in a group G with the operation * , there is only one identity and inverse. (6 marks)

d) State whether the following permutation is odd or even

$(13245)(679)(1011)$ (4 marks)