MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR APRIL SESSION EXAMINATION FOR BACHELOR OF EDUCATION

SMA 260: PROBABILITY AND STATISTICS

DATE: 25/8/2017 **TIME:**

INSTRUCTIONS

*Answer Question **ONE** which is compulsory and any other **TWO** Questions*

QUESTION ONE (30 MARKS)

(a) By citing examples differentiate between discrete and continuous random variable (4 marks)

(b) (i) Highlight three properties of a normal distribution (3 marks)

(ii) A process yields 10% defective items. If 100 items are randomly selected from the process, what is the probability that the number of defective exceeds 13? (5 marks)

(c) The average number of radioactive particles passing through a counter during 1 millisecond in a laboratory experiment is 4. What is the probability that 6 particles enter the counter in a given millisecond? (4 marks)

(d) Given that $F(x)$ is the CDF of a poison distribution with parameter λ and that $F(2) = 2F(1)$. Determine the value of λ (5 marks)

(e) Suppose the unit rate of receiving calls at an exchange board is 20 calls per hour. Determine in three hour interval;

i. The expected number of calls and its variance

ii. The probability of receiving exactly 20 calls (4 marks)

(f) If X is a hypergeometric random variable, show that $E(x)=\left(\frac{A}{N}\right)n$ (5 marks)

QUESTION TWO (20 MARKS)

(a) The *pdf* of a random variable X is given by

$$f(x)=\begin{cases}\frac{1}{3} & ,-1<x<2\\ 0 & otherwise\end{cases}$$

Calculate $E(x^2)$, $E(x^3)$ and μ_3 (10 marks)

(b) A pair of fair dice is tossed to obtain equiprobable space Ω consisting of 36 order pairs of numbers between 1 and 6. Determine the $f(x)$ and $F(x)$, represent $F(x)$ in a graph (7 marks)

(c) Cite three real life examples that follow a Poisson distribution (3 marks)

QUESTION THREE (20 MARKS)

(a) Given that X is a random variable with *pdf*

$$f(x)=\begin{cases}\frac{1}{2\sqrt{x}} & 1<x<4\\ 0 & elsewhere\end{cases}$$

Determine

i. The quartile deviation of the random distribution

ii. The 30th percentile of the random distribution (6 marks)

(b) A multiple-choice quiz has 200 questions each with four possible answers of which only one is the correct answer. What is the probability that sheer guesswork yields from 25 to 30 correct answers for 80 of the 200 problems about which the student has no knowledge? (6 marks)

(c) If X is the number heads that show up after tossing a fair coin twice, determine the mgf_x, $E(x)$ and $Var(x)$ (8 marks)

QUESTION FOUR (20 MARKS)

(a) If the prob. that an individual suffers a bad reaction from a certain injection is 0.001, determine the prob. that out of 2000 individuals, more than 2 individuals will suffer a bad reaction. (6 marks)

(b) A variable X has a hypergeometric distribution with parameters $A = 10$ and $B = 30$, for what value of n , will $Var(X)$ be maximum (8 marks)

The lifespan of bulbs manufactured by a certain company is normally distributed with $\mu = 2040$ hours and $\sigma = 60$ hours. This is based on an experiment of 20,000 bulbs. Based on the experiment determine the

i. Number of bulbs with a lifespan of 1,960 to 2,150 hours
ii. Probability that a bulb picked at random would last for more than 2,200 hours (6 marks)

QUESTION FIVE (20 MARKS)

(a) Highlight two properties of a binomial distribution (2 marks)

(b) Of a large number of mass-produced articles, one-tenth is defective. Find the probabilities that a random sample of 20 will obtain

(i) exactly two defective articles;
(ii) at least two defective articles (5 marks)

(c) Ships arrive in a harbour at a mean rate of two per hour. Suppose that this situation can be described by a Poisson distribution. Find the probabilities for a 30-minute period that

(i) No ships arrive;
(ii) Three ships arrive (5 marks)

(d) The diameter of an electric cable (x) is assumed to be continuous random variable with such that $f(x) = 6x(1-x)$ for $0 < x < 1$ otherwise it is zero.

i. Confirm that $f(x)$ is a pdf
ii. Determine a number b such that $p(x < b) = p(x > b)$ (8 marks)