



MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING

BACHELOR OF SCIENCE IN CIVIL ENGINEERING

SMA 404: COMPLEX ANALYSIS II

DATE: 27/7/2017

TIME: 11:00 – 1:00 PM

INSTRUCTIONS

Answer question ONE (Compulsory) and any other TWO questions

QUESTION ONE- COMPULSORY (30 MARKS)

- a) Evaluate $\lim_{z \rightarrow i} 2 \left(\frac{z^2 + iz + 2}{z - i} \right)$ (4 marks)
- b) Explain the nature of the transformation $w = z^2$ (5 marks)
- c) Find the image of a square $O(0,0)$, $A(1,0)$, $B(1,1)$ and $C(0,1)$ under transformation $T(z) = 3iz + 2 - 5i$ (6 marks)
- d) Determine whether $f(x)$ is analytic given that $f(x) = (x + \alpha y)^2 + 2i(x - \alpha y)$ for α been real and constant. (5 marks)
- e) Find the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at all its poles in the finite plane. (10 marks)

QUESTION TWO (20 MARKS)

- a) Calculate the residues for $f(z) = \frac{1}{z(z+1)}$. (5 marks)
- b) Find the image of the point $(4,3)$ on z -plane under the transformation $w = 2z^2 + 3$ (5 marks)
- c) Investigate for convergence or divergence of the series $\sum_{n=0}^{\infty} \frac{(z+3-i)^n}{4^n (n+1)^3}$ (10 marks)

QUESTION THREE (20 MARKS)

- a) Find the Laurent expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ for $1 < |z| < 2$ (6 marks)
- b) Solve $z^4 = i$ (7 marks)
- c) Show that $f_1(z) = \sum_{n=0}^{\infty} \left(\frac{z-i}{1-i} \right)^n$ and $f_2(z) = \sum_{n=0}^{\infty} z^n$ are analytic continuation of each other. Indicate the region on the Argand diagram (7 marks)

QUESTION FOUR (20 MARKS)

- a) Find the fixed points of the transformation $T(z) = \frac{3iz-5}{z-2z}$ (6 marks)
- b) Find the Mobius transformation which maps the points $z_1 = i$, $z_2 = 0$ and $z_3 = 1$ onto $w_1 = -1$, $w_2 = \infty$ and $w_3 = -i$ respectively. (7 marks)
- c) Consider the lines $l_1 : x = 2$ and $l_2 : y = 2$ meeting at $z = z_0 = (2,2)$. Discuss conformality of $T(z = z^2)$ at z_0 . (7 marks)

QUESTION FIVE (20 MARKS)

- a) Test if the following function satisfies the Schwartz-Reflection principle $f(z) = z^2$ (3 marks)
- b) Find the image of the transformation $T(z) = z^2$ (7 marks)
- c) Show that $\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2 + 1)^2} dx = \frac{2\pi}{e^3}$. (10 marks)