MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR DEGREE IN

BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING

BACHELOR OF SCIENCE IN BUILDING AND CIVIL ENGINEERING

BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

ECU106: ENGINEERING MATHEMATICS III

DATE: 15/6/2017 **TIME: 8:30 – 10:30 AM**

INSTRUCTION TO CANDIDATES:

ANSWER QUESTION ONE AND ANY TWO OTHER QUESTIONS.

QUESTION ONE (COMPULSORY) (30MARKS)

a) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 4 & 1 & 2 \end{bmatrix}$. Find the determinant of the matrix A. (3 marks)

b) Determine the value of the constant a such that the vectors $2i - j + k,\ i + 2j - 3k,\ \ and\ \ 3i + aj + 5k$ are coplanar (4 marks)

c) Find the equation of the plane determined by the points $A(2,-1,1),\ B(3,2,-1)$ and $C(-1,3,2)$. Also find the coordinates of the point where this line meets the plane $z = 0$ (4 marks)

d) If $\vec{A} = t^2\,\hat{i} - t\,\hat{j} + (2t-1)\hat{k},\ \ and\ \ \vec{B} = (2t-3)\hat{i} + \hat{j} - t\,\hat{k}$, evaluate $\frac{d}{dt}\left[\vec{A} \times \frac{d\vec{B}}{dt}\right]$ (4 marks)

e) Use Cramer's rule to solve

$$2x+3y-z=1$$
$$3x+5y+2z=8$$
$$x-2y-3z=-1$$

(5 marks)

f) Prove that the diagonals of a parallelogram bisect each other (5 marks)

g) Express $w=(-7,7,11)$ as a linear combination of the vectors $v_1=(1,2,1)$, $v_2=(-4,-1,2)$ and $v_3=(-3,1,3)$ (5 marks)

QUESTION TWO (20 MARKS)

a) Find the volume of the parallelepiped whose edges are represented by

$\vec{A}=2\hat{i}-3\hat{j}+4\hat{k}$, $\vec{B}=\hat{i}+2\hat{j}-\hat{k}$ *and* $\vec{C}=3\hat{i}-\hat{j}+2\hat{k}$ (5 marks)

b) Find the inverse of the following matrices

$$A=\begin{bmatrix}4 & 0 & 1\\ 2 & 2 & 0\\ 3 & 1 & 1\end{bmatrix}$$

(7 marks)

c) Determine the equation of the plane through $(-1,2,3)$ and perpendicular to the planes

$$2x-3y+4z=1$$
$$3x-5y+2z=3$$

(8 marks)

QUESTION THREE (20 MARKS)

a) Transpose the following matrix

$$\begin{bmatrix}5 & -7 & 3\\ 4 & 2 & 0\\ 3 & 4 & 7\end{bmatrix}$$

(3 marks)

b) Given that $A=5\hat{i}-2\hat{j}+3\hat{k}$, $\vec{B}=3\hat{i}+\hat{j}-2\hat{k}$ *and* $\vec{C}=\hat{i}-3\hat{j}+4\hat{k}$. Determine $\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)$

(5 marks)

c) Find the acute angle between the lines

$$\frac{x-1}{5}=\frac{y+2}{4}=\frac{z}{6} \text{ and } \frac{x+3}{3}=\frac{y-2}{7}=\frac{z+1}{-1}$$

(5 marks)

d) If $B=|b_{ij}|$ is obtained from $A=|a_{ij}|$ by adding to each element of the r^{th} row (column) of A a constant C times the corresponding element of the s^{th} row (column) $r\neq s$ of A, then $\det B=\det A$. Prove. (7 marks)

QUESTION FOUR (20 MARKS)

a) Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$.

Compute

i $|A|$ (2 marks)

ii $(adjA)A$ and $A(adjA)$ (3 marks)

iii $adjA$ (5 marks)

b) Calculate the value of k for the vectors $u = (1, k)$ and $v = (-4, k)$ knowing that they are orthogonal. (5 marks)

c) Find the rank of the following matrix

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & -3 & 2 \\ 2 & 0 & 3 \end{bmatrix}$$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Find the angle between the given two vectors $a = i + 4j + 8k$ and $b = 5i + 4j + 3k$

(5 marks)

b) Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ find $(A+B)^2 - (A^2 + 2AB + B^2)$ (5 marks)

c) Solve by inverse matrix method

$$x_1 + 3x_2 + 2x_3 = 3$$
$$2x_1 + 4x_2 + 2x_3 = 8$$
$$x_1 + 2x_2 - x_3 = 10$$
(10 marks)