



MACHAKOS UNIVERSITY

University Examinations for 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONIC ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 200: ENGINEERING MATHEMATICS V

DATE: 6/12/2021

TIME: 11:00 – 1:00 PM

INSTRUCTIONS: Attempt Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Differentiate between an Indefinite and a definite integral. (2 marks)

b) Determine the following integrals:

i. $\int \frac{x^3 + 5x^2 - 4}{x^2} dx$ (2 marks)

ii. $\int_0^2 \frac{x^2}{x^3 - 1} dx$ (4 marks)

iii. $\int \frac{dx}{x^2 + 3}$ (4 marks)

c) Find the number c that satisfies the mean value theorem of integral calculus for the

function $y = \frac{1}{2x + 1}$ over the interval $[0, 1]$. (3 marks)

d) Determine the double integral $\iint x^2 y dy dx$. (2 marks)

e) Determine $\lim_{x \rightarrow 1} \left\{ \frac{x^2 + 3x - 4}{x^2 - 7x + 6} \right\}$ (2 marks)

- f) Evaluate the area bounded by the curve $y = \sin x$, the x-axis and the lines $x = 0$ and $x = \frac{3\pi}{2}$. (4 marks)
- g) Given $Ae^x + Be^{-x} = 4 \cosh x - 5 \sinh x$. Determine the values of A and B. (3 marks)
- h) The region bounded by the curve $y = x^2$ and the line $y = x$ is rotated about the y-axis. Calculate the volume of the solid of revolution generated. (4 marks)

QUESTION TWO (20 MARKS)

- a) Find the area below $f(x) = -x^2 + 4x + 3$ and above $g(x) = -x^3 + 7x^2 - 10x + 5$ over the interval $1 \leq x \leq 2$. (3 marks)
- b) Suppose a population of bacteria grows at a rate of $P'(t) = 0.1t (\ln t)^2$. Find a formula for $P(t)$. (5 marks)
- c) A yeast culture is growing at a rate of $\frac{d\mu}{dt} = 0.3e^{0.2t}$, where t is the time in hours, $\mu(t)$ is the mass in grams. At time $t = 0$ the mass was 0.1 grams.
- Use Simpson's Rule with $r = 6$ to estimate the mass after 3 hours. (5 marks)
 - Estimate the absolute value of the maximum error that can occur when growing the culture after 3 hours. (3 marks)
- d) Estimate the absolute value of the maximum error that can occur when approximating: $\int_1^{3/2} \frac{dx}{x}$, $r = 4$ using Simpsons Rule. (4 marks)

QUESTION THREE (20 MARKS)

- a) A particle starts in motion with velocity of $10m/s^2$ and its acceleration after time t is given by $a = -2t + 3$. Find the distance travelled when the particle first comes to rest. (8 marks)
- b) Using trapezoidal rule estimate $\int_1^4 \ln x dx$. Convert to four decimal places and take $r = 6$. (4 marks)
- c) Evaluate $\int \frac{dx}{4x^2 + 4x + 12}$ (6 marks)
- d) Evaluate $\sinh 5.4$ correct to 4 significant figures. (2 marks)

QUESTION FOUR (20 MARKS)

- a) Suppose a sky diver leaps from a hovering helicopter and falls in a straight line. He falls at a terminal velocity of 80m/s for 19 seconds, at which time he opens his parachute. The velocity decreases linearly to 6m/s over a two-second period and then remains constant until he reaches the ground at $t = 40$ seconds. The motion is described by the velocity function:

$$V(t) = \begin{cases} 80 & \text{if } 0 \leq t \leq 19 \\ 783 - 37t & \text{if } 19 \leq t \leq 21 \\ 9 & \text{if } 21 \leq t \leq 40 \end{cases}$$

Determine the altitude from which the sky diver jumped. (3 marks)

- b) Evaluate $\int \frac{x^2 + 3x - 8}{x^2 + 2x - 8} dx$ (5 marks)
- c) Use tabular integration to find $\int x^2 \sqrt{x-1} dx$ (3 marks)
- d) Use Euler's method to obtain a numerical solution of the differential equation $\frac{dy}{dx} = 3(1+x) - y$ given the initial conditions that $x = 1$ when $y = 4$, for the range $x = 1.0$ to $x = 2.0$ with intervals of 0.2. (6 marks)
- e) Prove the hyperbolic identity $\cosh^2 x - \sinh^2 x = 1$. (3 marks)

QUESTION FIVE (20 MARKS)

- a) Express $\frac{x^3 + 2x^2 + x - 1}{x^2 + x - 2}$ in partial fractions and use your result to find the integral $\int \frac{x^3 + 2x^2 + x - 1}{x^2 + x - 2} dx$ (6 marks)
- b) Use the substitution $t = \tan \frac{x}{2}$ to evaluate $\int_{\pi/2}^{\pi} \frac{dx}{1 - \cos x}$ (5 marks)
- c) Determine the power series for $2 \cosh\left(\frac{\theta}{2}\right) - \sinh 2\theta$ as far as the term θ^5 . (3 marks)
- d) What is the coefficient of $x^{12}y^{13}$ in the expansion $(2x - 3y)^{25}$ (2 marks)
- e) Expand $\ln(1+x)$ in five terms using Maclaurin's Series. (4 marks)