MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

BACHELOR OF ARTS

SMA 230: VECTOR ANALYSIS I

DATE: 27/9/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

Attempt question ***one (compulsory***) and any other ***two questions***.

Question 1(30mks)

a.) Given that $A = 5\hat{i} - 2\hat{j} + 3\hat{k},\ \vec{B} = 3\hat{i} + \hat{j} - 2\hat{k}\ \ and\ \ \vec{C} = \hat{i} - 3\hat{j} + 4\hat{k}$. Determine ;
$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)$ (4mks)

b.) If $\vec{A} = t^2\,\hat{i} - t\,\hat{j} + (2t-1)\hat{k},\ \ and\ \ \vec{B} = (2t-3)\hat{i} + \hat{j} - t\,\hat{k}$, evaluate $\frac{d}{dt}\left[\vec{A}\times\frac{d\vec{B}}{dt}\right]$

(4mks)

c.) Given that $\vec{F}(t)$ has a constant magnitude, show that $\vec{F}(t)$ and $\frac{d\vec{F}}{dt}$ are perpendicular provided that $\left|\frac{d\vec{F}}{dt}\right| \neq 0$ (5mks)

d.) Determine the value of the constant a such that the vectors
$2i - j + k,\ \ i + 2j - 3k,\ \ and\ \ 3i + aj + 5k$ are coplanar (3mks)

e.) Evaluate the direction from the point $(1,1,\ 0)$ which gives the greatest rate of increase of the function $\phi = (x\ \ +3y)^2 + (2y-z)^2$ (4mks)

f.) Express the heat equation $u_t = k\nabla^2 u$ in spherical coordinates. (5mks)

g.) Evaluate $\iint (\nabla \times \vec{F}) \cdot \hat{n}\, ds$, where $\vec{F} = (x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xz + z^2)\hat{k}$ and S is the surface hemisphere $x^2 + y^2 + z^2 = 16$ above the $x - y$ *plane* . (5mks)

Question 2(20mks)

a.) Given that $\vec{F} = x^2\hat{i} - xy\hat{j}$ .Evaluate $\int \vec{F} \cdot dR$ from $(0,0)$ to $(1,1)$ along.

i.) The straight line $(0,0)$ to $(1,1)$ (5mks)

ii.) The parabola $y^2 = x$ (3mks)

b.) Determine the surface integral $\iint_S A \cdot nds$ where $A = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first quadrant (7mks)

c.) Evaluate $\nabla^2(\ln r)$ (5mks)

Question 3 (20mks)

a.) Evaluate $\iiint_V \vec{F}dv$ where $\vec{F} = 2xz\hat{i} - x\hat{j} + y\hat{k}$ and v is the region bounded by the surfaces $x = 0$, $y = 6$, $z = x^2$ and $z = 4$
(7mks)

b.) Consider the force field $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2 k$;

i. Show that the force field is conservative (4mks)

ii. Calculate the scalar potential (5mks)

iii. Determine work done in moving an object in the field from $(1,-2,1)$ *and* $(3,1,4)$ (4mks)

Question 4(20mks)

a.) Prove that $\vec{r} = e^{-t}[c_1 \cos 2t + c_2 \sin 2t]$ satisfies the differential equation $\frac{d^2\vec{r}}{dt^2} + 2\frac{d\vec{r}}{dt} + 5\vec{r} = 0$ where c_1 *and* c_2 are constants. (6mks)

b.) Given the space curve $x = t$, $y = t^2$, $z = \frac{2}{3}t^3$. Calculate ;

i.) The curvature K (7mks)

ii.) The torsion τ (7mks)

iii.)

Question 5(20mks)

a.) Calculate the volume of the parallelepiped whose edges are represented by $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$ *and* $\vec{C} = 3\hat{i} - \hat{j} + 2\hat{k}$ (5mks)

b.) Using Greens theorem, calculate the area enclosed by the ellipse $x = a\cos\theta,\ y = b\sin\theta\ ,\ 0 \le \theta \le 2\pi$ (5mks)

c.) A surface consists of that part of the cylinder $x^2 + y^2 = 9$ between $z = 0$ and $z = 4$ for $y \ge 0$ and the two semicircles of radius 3 units in the planes $z = 0$ and $z = 4$. Given that $\vec{F} = z\hat{i} + xy\hat{j} + xz\hat{k}$, evaluate $\iint_s curl\ \vec{F} \cdot ds$ over the surface (10mks)