MACHAKOS UNIVERSITY

ISO 9001:2008 CERTIFIED

SCHOOL OF PURE AND APPLIED SCIENCES FOR THE BACHELOR'S DEGREE

UNIVERSITY SUPPLEMENTARY EXAMINATIONS

SMA 104: CALCULUS I

DEPARTMENT OF MATHEMATICS AND STATISTICS.

Time 2hours

Attempt question ***one (compulsory)*** and any other ***two questions***.

Question 1(30marks)

a.) Determine the inverse of the function $f(x)=\sqrt{4-x}$ (3marks)

b.) Evaluate

$$\lim_{x \to 2} \frac{x-2}{x^2-x-2}$$ (3marks)

c.) Consider the function $h(x)$ such that $h: x \to x^2$ defined on the set $\Re$ by the elements $x_1 = -2$, $x_2 = 2$. Show that $h(x)$ is a surjective function. (3marks)

d.) Prove the continuity of the function $f(x)=3x+1$ at the point $x=1$ (3marks)

e.) Approximate $\sin 29^0$ using small changes given that $1^0 = 0.0175^c$ (3marks)

f.) Calculate $\frac{d^2y}{dx^2}$ for the parametric equations $x=2t$, $y= t^2-1$ (3marks)

g.) Evaluate $\frac{dy}{dx}$ for;

i.) $y=\frac{\sqrt{\cos x}}{x^2 \sin x}$ (4marks)

ii.) $f(x)=x^2y+4y=8$ *at the point* $(2,1)$ (4marks)

iii.) $f(x)=\left(12x^3-4x^2-x+1\right)^4$ (4marks)

Question 2(20marks)

Consider the function $f(x)=x^5-5x^3$

a) Determine using the second derivative

i.) Local maximum point (3marks)

ii.) Local minimum point (3marks)

b) Work out the coordinate of the points of inflection of $f(x)$ (4marks)

c) Discuss the concavity of $f(x)$ (5marks)

d) Sketch the graph of $f(x)$ (5marks)

Question 3(20marks)

a.) Given that $x=\sin t$, $y=5\cos t$ determine $\frac{dy}{dx}$ when $t=\frac{\pi}{3}$ (4marks)

b.) Determine $\frac{dy}{dx}$

iii.) $y=\ln\left[\frac{x\sqrt{x+5}}{(x-1)^3}\right]$ (4marks)

iv.) $y=\sec^3 x$ (4marks)

v.) $y=x^{x^{x^{\infty}}}$ (4marks)

vi.) $y=\sinh^{-1} x$ (4marks)

Question 4 (20marks)

a.) A closed rectangle box is made of very thin sheet metal and its length is three times its width. If the volume of the box is $288cm^3$;

i) Show that its surfsce area is equal to $s=\frac{768}{x}+6x^2$ where $x\ cm$ is the width of the box. (2marks)

ii) Determine the dimentions of the box of least surface area (2marks)

iii) State the value of this surface area. (2marks)

b.) Given that $y=4^{3x}-5e^{-3x}$ show that $\frac{d^2y}{dx^2}=9y$ (4marks)

c.) Given that $f(x)=(\cos x-2^x)(\ln x+3\sin x)$, use the product rule to obtain Given that $f'(x)$ (5marks) d.) Given that $f(x)=\dfrac{ax+b}{cx+d}$, use the quotient rule to show that $f'(x)=\dfrac{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{(cx+d)^2}$

(5marks)

Question 5(20marks)

a.) Given two functions: $f(x)=3x-2,\ g(x)=8-9x$, determine;

i) $g[f(x)]$ (3marks)

ii) $f^{-1}(x)$

(3marks)

b.) Consider the Folium of Descartes equation ; $x^3+y^3-6xy=0$

i) Determine its tangent slope at the point $\left(\dfrac{4}{3},\dfrac{8}{3}\right)$ (3marks)

ii) Calculate its normal line equation at the point $\left(\dfrac{4}{3},\dfrac{8}{3}\right)$ (3marks)

c.) A particle is moving in a straight line and its distance s metres from a fixed point in the line after t seconds is given by the equation $s=12t-15t^2+4t^3$. Determine:

i) The velocity of the particle after 3 seconds (2marks)

ii) Acceleration of the particle after 3 seconds (2marks)

iii) Distance travelled between the two times when the velocity is instantaneously zero. (4marks)