



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 103: ANALYTICAL GEOMETRY

DATE: 2/3/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Determine the normal equation of the line passing through points $(3,5)$ and $(4,9)$ (3 marks)
- b) Find the equation of the tangent line to the circle $x^2 + y^2 + 2x - 4y - 4 = 0$ at the point $(-1,-1)$ (5 marks)
- c) Solve $\cos(x + 20) - \cos(x + 80) = 0.5$ (4 marks)
- d) A parabola $y = ax^2 + bx + c$ passes through the points $(1,0)$, $(2,2)$, and $(3,10)$. Determine its equation. (5 marks)
- e) Determine the Cartesian equation for the curve $r \cos\left(\theta - \frac{\pi}{3}\right) = 3$ (4 marks)
- f) Consider the equation $x^2 + 4x = y$;
- i. Express it in standard form (3 marks)
- ii. Determine the equation of directrix. (2 marks)
- g) Determine the polar equation of the hyperbola $x^2 - 4y^2 = 4$ (4 marks)

QUESTION TWO (20 MARKS)

- a) Determine an equation of the circle that contains the point (3,1) and passes through the points of intersection of the two circles $x^2 + y^2 - x - y - 2 = 0$ and $x^2 + y^2 + 4x - 4y - 8 = 0$ (8 marks)
- b) Calculate the equation of the circle which is tangent to the x-axis and concentric with the circle $2x^2 + 2y^2 - 11x + 6y - 8 = 0$ (5 marks)
- c) Determine the polar equation of the circle whose center has the Cartesian coordinates $C(5,8)$ and radius $r = 10\text{cm}$ (4 marks)
- d) Calculate the distance from the point (5,8) to the circle $x^2 + y^2 + 2x + 6y + 3 = 0$ (3 marks)

QUESTION THREE (20 MARKS)

- a) If $\tan \alpha = \frac{1}{5}$, $\tan \beta = \frac{4}{19}$ and $\tan \gamma = \frac{2}{5}$ find $\alpha + \beta + \gamma$ without evaluating the values of α, β, γ (5 marks)
- b) Find the values of x between 0° and 360° satisfying the equation $12 \cos x - 5 \sin x + 3 = 0$ (8 marks)
- c) If A and B are acute angles such that $\sin A = \frac{\sqrt{5}}{5}$ and $\tan B = \frac{1}{3}$ find the value of $(A + B)$ by means of addition formula for $\sin (A + B)$ (4 marks)
- d) Find the polar coordinates of the point whose Cartesian coordinates are $(-5,12)$ (3 marks)

QUESTION FOUR (20 MARKS)

- a) Analyze the curve whose equation is expressed as $9y^2 + 4x^2 + 18y + 8x - 23 = 0$ (10 marks)
- b) Determine the equation of the tangent to the parabola $x^2 - 22x + 24y + 169 = 0$ at the point whose focal distance is 11 units (10 marks)

QUESTION FIVE (20 MARKS)

- a) Sketch the curve and label axes, foci and vertices described by the equation

$$\frac{(x-1)^2}{4} - \frac{(y+2)^2}{9} = 1 \quad (8 \text{ marks})$$

- b) Find the tangent of the circle $x^2 + y^2 = 5$ from the point $(8,3)$ (5 marks)

- c) Find the equation of the straight line passing through $(1,2)$ and perpendicular to the line $x + y + 7 = 0$ (4 marks)

- d) Show that $\cos \theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 4 \cos \theta \cos 2\theta \cos 4\theta$ (3 marks)