MACHAKOS UNIVERSITY

DEPARTMENT OF MATHEMATICS AND STATISTICS

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE
BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING
BACHELOR OF SCIENCE IN MATHEMATICS
BACHELOR OF EDUCATION

UNIVERSITY SUPPLEMENTARY EXAMINATIONS

SMA 464 : DESIGN AND ANALYSIS OF EXPERIMENTS

Instructions to the Candidate:

1. *Answer **Question 1** and any other **two** questions.*
2. *Out of the **three** questions answered, each question must start on a new page.*
3. *You need a Scientific Calculator and Statistical Tables for this paper.*

1. (a) Explain the term *blocking* or *error control* as a technique of error control in the design of field experiments, illustrating with an example from a real life environment. (4 *marks*)

   (b) Differentiate between an *experimental group* and a *control group* as used in the design of experiments, illustrating with an example from a real life situation. (4 *marks*)

   (c) Given a one-way classification ANOVA with the model given by $y_{ij} = \mu + t_i + e_{ij}$, and taking the various sums of squares, prove the of the following: (6 *marks*)

   (i) Treatment sum of squares: $$SS_t = \sum_{i=1}^{k} \sum_{j=1}^{n} \left(\bar{y}_{i.} - \bar{y}_{..}\right)^2 = \sum_{i=1}^{k} \frac{{T_i}^2}{n_i} - \frac{G^2}{N}$$

   (ii) Error sum of squares: $$SS_e = \sum_{i=1}^{k} \sum_{j=1}^{n} \left(y_{ij} - \bar{y}_{i.}\right)^2 = \sum_{i=1}^{k} \sum_{j=1}^{n} {y_{ij}}^2 - \sum_{i=1}^{k} \frac{{T_i}^2}{n_i}$$

   (d) An animal breeding researcher carried out a study on breeds of dairy cows on milk production. Three breeds of dairy cows were tested: local breed, cross breed and exotic breed. A random sample of 8 dairy cows was taken from each breed. The 24 cows were then kept under identical conditions for a period of one year. The mean monthly production of milk for each of the 24 cows was recorded in litres. After statistical analysis, he derived the following ANOVA table.

Source of Variation	Sum of Squares SS	Degrees of Freedom	Mean Sum of Squares - MSS	Variance Ratio F	P–value
Breed	849.0000				0.00022626823
Error	693.5000				
Total					

Given the treatment means: Local $\bar{y}_{1.}$ = 70.50 , Cross $\bar{y}_{2.}$ = 75.00 , Exotic $\bar{y}_{3.}$ = 84.75

(i) Complete the ANOVA table above, and hence using the test statistic F, evaluate whether there is a significant difference in milk production between the breeds. (4 *marks*)

(iii) Compute the critical difference between the breeds, and hence identify which pair of animal breeds differ significantly. (4 *marks*)

(d) A study was carried out to assess if there is a difference in academic performance in Design and Analysis of Experiments between three B.Sc. courses – K63, S10, and E35. A random sample of 10 students was taken from each course. The marks for the 30 students were then recorded as shown in the table below:

Course	Observations (marks scored)									
K63	56	64	50	36	54	42	32	48	56	64
S10	40	74	45	48	65	68	42	74	40	74
E35	84	64	65	72	56	74	70	55	84	64

Using a complete randomised design (CRD), evaluate whether there is a significant difference between the three courses in academic performance of the students. Test at both the 5% level and 1% level of significance. (8 *marks*)

2. (a) With the aid of a suitable example, differentiate between the terms *fixed effects model* and *random effects model* as used in the design and analysis of experiments. (6 *marks*)

(b) Given a one-way ANOVA with the model given by $y_{ij} = \mu + t_i + e_{ij}$, then under the fixed effects model, show that:

(i) the mean sum of squares due to error (MSS_e) is an unbiased estimator of the error variance σ_e^2 ; (6 *marks*)

(ii) the mean sum of squares due to treatment (MSS_t) is a biased estimator of the error variance σ_e^2 . (8 *marks*)

3. An animal breeding researcher carried out a study on the effect of exotic breeds of dairy cows on milk production. Three breeds of dairy cows were tested: local breed, cross breed and exotic breed. A random sample of 8 dairy cows was taken from each breed. The 24 cows were then kept under the same conditions and given the same feedstuff for a period of one year. The mean monthly production of milk for each of the 24 cows was recorded in litres as shown in the table below:

Cow Breed	Observations (milk yield in litres)							
Local	64	70	66	68	60	62	78	72
Cross	62	76	84	72	78	68	80	74
Exotic	84	78	92	86	90	84	94	88

Using a complete randomised design (CRD) assuming the linear additive model $y_{ij} = \mu + t_i + e_{ij}$, and using a 5% level of significance:

(i) State *three* assumptions in the model. (3 *marks*)

(ii) Evaluate whether there is a significant difference between the breeds (treatments) in the milk production among the dairy cows. (9 *marks*)

(iii) Compute the critical difference between the breeds of cows, and identify which pair of breeds has a statistically significant difference. (6 *marks*)

(iv) Which breed of dairy cows would you recommend, justifying your choice? (2 *marks*)

4. A medical researcher carried out a study on the effect of anaesthetic drugs on patients in a theatre to assess the duration it takes for a patient to become unconscious. Four types of drugs were tested with 5 different drug dosages in millilitres. A random sample of 20 patients was taken and 5 patients randomly assigned to each drug. Each dosage was randomly assigned to 4 patients with each patient given a different drug. The duration in minutes a patient took to fall unconscious was recorded as shown in the table below:

Dosage	10 ml	15 ml	20 ml	25 ml	30 ml
Drug A	48	50	40	56	50
Drug B	52	48	30	40	46
Drug C	45	34	32	30	42
Drug D	20	26	18	20	38

(a) Using a randomised block design (RBD) with drugs as treatments and dosage as blocks, and two-way ANOVA with the model $y_{ij} = \mu + t_i + b_j + e_{ij}$, and using a 5% level of significance:

(i) Evaluate whether there is a significant difference between the drugs in their effectiveness in treating malaria. (8 *marks*)

(ii) Evaluate whether there is a significant difference between the dosages of the drugs in the effectiveness in treating malaria. (8 *marks*)

(b) Based on the findings in (a) above, recommend the best drug and the best dosage justifying your. (4 *marks*)

5. (a) Given a two-way ANOVA with its model $y_{ij} = \mu + t_i + b_j + e_{ij}$ and its corresponding total sum of squares given by $SS_T = \sum_{i=1}^{k} \sum_{j=1}^{n} \left(y_{ij} - \bar{y}_{..} \right)^2$, decompose the total sum of squares into its various components, and hence show that $SS_T = SS_t + SS_b + SS_e$ (4 *marks*)

(b) An automotive engineer carried out a study on the efficiency of break fluids on the breaking distance of a vehicle. In the study, 4 types of break fluids were considered. A sample of 4 types of vehicle engines was taken and 4 fluid transmission systems taken and tested on a random sample of 16 vehicles. Taking the break fluids as the treatments, the rows as the type of engine, and columns as the fluid transmission system, the design forms a 4 X 4 Latin square design. The yield in terms of the distance in metres a vehicle moving at a speed of 50 km per hour takes to stop immediately after the break pedal is pressed was recorded as shown in the table below:

	Column 1	Column 2	Column 3	Column 4
Row 1	T_3 = 35	T_2 = 14	T_1 = 10	T_4 = 45
Row 2	T_2 = 12	T_4 = 35	T_3 = 20	T_1 = 12
Row 3	T_1 = 10	T_3 = 15	T_4 = 34	T_2 = 14
Row 4	T_4 = 16	T_1 = 12	T_2 = 10	T_3 = 10

Using an ANOVA model given by $y_{ijk} = \mu + r_i + c_j + t_k + e_{ijk}$, and using a 5% level of significance:

(i) Evaluate whether there is a significant difference in the maize yield:

- between the break fluids;
- between the types of engine;
- between the fluid transmission systems. (14 *marks*)

(ii) What conclusions would you draw from the findings in (a)? (2 *marks*)