MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

SMA 463: TIME SERIES ANALYSIS

DATE: 8/5/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Using mathematical notations, where necessary, define the following terms as used in time series analysis: -

i) a time series (2 marks)

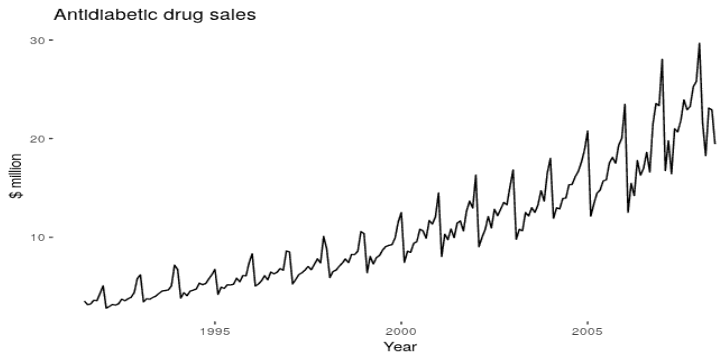
ii) IID noise (2 marks)

iii) ARMA(p,q) process (3 marks)

b) What is the main purpose of time series analysis? (2 marks)

c) State the four components of a time series. (4 marks)

d) The graph below shows the sales of antibiotic drugs over a period of 10 years.



What statistical conclusions can you draw from the graph, based on time series analysis? (2 marks)

e) i) Briefly describe the process of calculating moving averages of period m. (3 marks)

ii) Using the data given below, calculate the 5-point moving totals and moving averages.

t	1	2	3	4	5	6	7	8	9	10
X_t	12	10	11	11	9	11	10	10	11	12

(6 marks)

f) Given an ARMA(p,q) process, show that

when p=0 we have an MA(q) process and (3 marks)

when q=0 we have an AR(p) (3 marks)

QUESTION TWO (20 MARKS)

a) Define stationarity. (2 marks)

b) Consider the series defined by the equation

$X_t = Y_t + \Theta Y_{t-1}$, for $t = 0, \pm 1, \pm 2, \ldots$

where $\{Y_t\} \sim WN(0, \sigma^2)$ and $|\Theta| < 1$ is a real-valued parameter.

Show that $\{X_t\}$ is stationary. (8 marks)

c) Define the term linear process as used in time series analysis. (3 marks)

d) Prove that, if $\frac{1}{2q+1}\sum_{j=-q}^{q} Y_{t-j} \approx 0$ (usually what happens if we choose q "large enough") this filter will allow a linear trend function $m_t = \beta_0 + \beta_1 t$ to pass without distortion. (7 marks)

QUESTION THREE (20 MARKS)

a) State two limitations of the method of moving averages. (2 marks)

b) The following data refers to the UK outward passenger movement by sea.

	Year 1				Year 2				Year 3			
Quarter	1	2	3	4	1	2	3	4	1	2	3	4
No. of passengers (millions)	2.2	5.0	7.9	3.2	2.9	5.2	8.2	3.8	3.2	5.8	9.1	4.1

Using least squares regression, calculate a trend component for each time point given.

(8 marks)

c) Let the random walk $\{S_t;\, t = 0, 1, 2, \ldots\}$ be obtained by summing up random variables. The random walk has zero mean and is obtained by defining

$S_0 = 0$ and

$S_t = X_1+X_2 + \ldots + X_t$ for $t = 1, 2, \ldots$

$\blacktriangledown\ S_t = S_t - S_{t-1} = X_t$

$\{X_t\} \sim IID\ (0, \sigma^2)$

Determine whether the random walk is stationary of not. (10 marks)

QUESTION FOUR (20 MARKS)

a) Define the terms

MA(q) and AR(p), as used in time series analysis. (4 marks)

b) Show that the MA(q) process given by

$X_t = Z_t + \Theta_1 X_{t-1} + \Theta_2 X_{t-2} + \ldots + \Theta_q X_{t-q}$ is a linear process. (4 marks)

c) Consider the series defined by the equation

$X_t = Y_t + \Theta Y_{t-1}, \qquad t = 0,\pm1,\pm2, \ldots$

where $\{Yt\}\sim WN(0,\sigma^2)$ and $|\theta|<1$ is a real-valued parameter. Determine the

i) autocovariance (ACVF) function (5 marks)

ii) autocorrelation (ACF) function. (4 marks)

d) Sketch the correlogram of the equation in (c) above. (3 marks)

QUESTION FIVE (20 MARKS)

a) Define the term causality as used in time series analysis. (3 marks)

b) Show that

(i) a MA(1) process with representation

$X_t = Z_t + \Theta_1 X_{t-1} + \Theta_2 X_{t-2} + \dots + \Theta_q X_{t-q}$ is causal (4 marks)

(ii) an AR(1) process define by $Xt = Z_t + ØX_{t-1}$ is causal (4 marks)

c) Given that the difference operator ∇ is defined by

$\blacktriangledown X_t = X_t - X_{t-1}$. Determine $\nabla^2 X_t$ (4 marks)

d) Define a correlogram. (2 marks)

e) State three conditions that are necessary for stationarity. (3 marks)