



MACHAKOS UNIVERSITY,

ECU 300: ENGINEERING MATHEMATICS IX

University Supplementary Examinations

Instructions: Answer Question (Compulsory) and any other 2 Questions.

Question one (30 marks)

- a) Find unit vectors normal to the surfaces $x^2 + y^2 - z^2 + 3 = 0$ and $xy - yz + zx - 10 = 0$ at the point (3, 2, 4) and hence find the angle between the two surfaces at that point. 4marks
- b) Prove that $\nabla^2 \left(\frac{1}{r} \right) = 0$ where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$ 5marks
- c) Prove the vector triple product $\mathbf{b} \times (\mathbf{c} \times \mathbf{a}) = (\mathbf{b} \cdot \mathbf{a})\mathbf{c} - (\mathbf{b} \cdot \mathbf{c})\mathbf{a}$ 4marks
- d) Determine the unit base vectors in the directions of the following vectors and determine whether the vectors are orthogonal:
 $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}, 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}, \mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ 4marks
- e) Find the directional derivative of $\phi = (x + 2y + z)^2 - (x - y - z)^2$ at the point (2,1,-1) in the direction of $\mathbf{A} = \mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$. 4marks
- f) Using Divergence theorem, evaluate $\iint_S \vec{A} \cdot \vec{n} \, ds$ where $\vec{A} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ taken over the region bounded by $x^2 + y^2 = 4, z=0$ to $z=3$. 4marks
- g) Given the space curve $x = 2t, y = t^2$ and $z = \frac{2}{3}t^3$ find;
(i) Curvature k
(ii) Torsion τ . 5marks

Question Two (20 marks)

a) Given that $\vec{R} = e^{-t}\mathbf{i} + \ln(t^2 + 1)\mathbf{j} + \tan t\mathbf{k}$, determine;

i) $\left| \frac{d\vec{R}}{dt} \right|$ at $t = 0$ 3marks

ii) $\left| \frac{d^2\vec{R}}{dt^2} \right|$ at $t = 0$ 3marks

b) A particle moves along the curve $x = 2t^2, y = t^2 - 4t$ and $z = 3t - 5$, determine the components of its velocity and acceleration at a time $t = 1$ in the direction of

$\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ 7marks

c) Given that $\vec{A} = x^2yz\mathbf{i} - 2xz^3\mathbf{j} + xz^2\mathbf{k}$ and $\vec{B} = 2z\mathbf{i} + y\mathbf{j} - x^2\mathbf{k}$, determine $\frac{\partial^2}{\partial x \partial y}(\vec{A} \times \vec{B})$ at $(1, 0, -2)$ 7marks

Question Three (20 marks)

a) If V is a scalar field such that $V = u^2vw^3$ and scale factors are $h_u = 1, h_v = 2$ and $h_w = 4$, determine $\nabla^2 V$ at the point $(2, 3, -1)$. 5marks

b) Prove the following Frenet-Serret formulae:

$$\frac{d\vec{B}}{ds} = -T\vec{N}$$

7marks

c) If $\phi = 2xyz^2$ and $\mathbf{F} = xy\mathbf{i} - z\mathbf{j} + x^2\mathbf{k}$ and C is the curve $x=t^2, y=2t$ and $z=t^3$ from $t=0$ to $t=1$; evaluate the line integrals:

i) $\int_C \phi dr$

ii) $\int_C \mathbf{F} \times d\mathbf{r}$ 8marks

Question Four (20 marks)

- a) Find the equation of the tangent plane to the plane surface $2xz^2 - 3xy - 4x = 7$ at the point $(2, -2, 3)$ 4marks
- b) Find the directional derivative $\phi = x^2yz + 4xz^2$ at $(1, -2, 1)$ in the direction $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ 4marks
- c) Verify Green's theorem in the plane for $\oint_c (xy + y^2)dx + x^2 dy$ where c is the closed curve of the region bounded by $y=x$ and $y=x^2$. 6marks
- d) Given that $\vec{v} = \vec{w} \times \vec{r}$, where $\vec{w} = w_1\mathbf{i} + w_2\mathbf{j} + w_3\mathbf{k}$ and $\vec{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, prove that $\vec{w} = \frac{1}{2} \text{curl } \vec{v}$. 6marks

Question Five (20 marks)

- a) Find the work done in moving a particle in a force field given by $\mathbf{F} = 3xy\mathbf{i} - 5z\mathbf{j} + 10x\mathbf{k}$ along the curve $x = t^2 + 1; y = 2t^2; z = t^3$ from $t=1$ to $t=2$. 6marks
- b) If $\mathbf{F} = 2xz\mathbf{i} - x\mathbf{j} + y^2\mathbf{k}$, evaluate $\iiint_V \vec{F} dV$ where V is the region bounded by the surfaces $x=0$, $y=0$, $y=6$, $z=x^2$ and $z=4$. 7marks
- c) Evaluate $\iint_S \vec{A} \cdot \vec{n} ds$ where $\mathbf{A} = z\mathbf{i} + x\mathbf{j} - 3zy^2$ and s is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z=0$ and $z=5$. 7marks