



MACHAKOS UNIVERSITY

University Examinations for 2018/2019 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

THIRD YEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF ECONOMICS

EES 300: MATHEMATICS FOR ECONOMISTS III

DATE: 27/9/2019

TIME: 8:30 – 10:30 AM

INSTRUCTIONS

Answer Question ONE and any other TWO questions

QUESTION ONE (COMPULSORY (30 MARKS))

a) The marginal costs of two firms are given by the following functions:

1) $C'(Q) = 15 + 20Q - 12Q^2$ $TC = 100$ when $Q = 0$

2). $C'(Q) = 100e^{0.2Q}$ $TC = 3000$ when $Q = 0$

i) Find the total cost function $C(Q)$ for each firm. [4 marks]

ii) Find the average cost function for each firm [4 marks]

iii) What is the fixed costs of each firm [2 marks]

b) Find the time path of capital $K(t)$ given the following rates of net investment flow functions

i) $I(t) = 5t^{1/2}$ $K(0) = 500$ [3 marks]

ii) $I(t) = 18t^{3/5}$ $K(0) = 30$ [3 marks]

iii) For each of (i) to (ii) above, find the amount of capital formation over the

interval $[0,5]$ [6 marks]

c) Find the producer surplus of the following

$Q = \sqrt{P - 6}$ given that $\bar{P} = 20$ [4 marks]

- d) Verify that the following differential equation is exact and solve the equations
 $8ytdy + (4y^2 + 3t)dt = 0$ [4 marks]

QUESTION TWO (20 MARKS)

- a) Derive general solution of the First order differential equations (FOLDE) [6 marks]
- b) Suppose you are given the following demand and supply functions
 $Q_d = \alpha - \beta P \quad (\alpha, \beta > 0)$
 $Q_s = -\gamma + \delta P \quad (\gamma, \delta > 0)$
- i) Assuming that the rate of change of price over time is directly proportional to the excess demand, find the time path $P(t)$ (general solution) [6 marks]
- ii) What is the inter-temporal equilibrium price [2 marks]
- iii) What is the market clearing equilibrium price [2 marks]
- iv) Does the market have a dynamically stable equilibrium price? Explain [4 marks]

QUESTION THREE (20 MARKS)

- a) Find the general and definite solution to the following differential equations
- i) $\frac{dy}{dt} + 4y = 10 \quad y(0) = 12$ [2 marks]
- ii) $\frac{dy}{dt} + 10y = 12 \quad y(0) = 20$ [2marks]
- iii) $\frac{dy}{dt} = 5y - 10 \quad y(0) = 3$ [2 marks]
- b) Find the integral of the following
- i) $\int \left(8x^2 e^{(x^3+10)} + \frac{4}{x^3} \right) dx \quad (x \neq 0)$ [2 marks]
- ii) $\int (1n x)^3 dx$ [2 marks]
- iii) $\int x(x^2 + 3)^{\frac{1}{2}} dx$ [2 marks]
- iv) $\int_2^4 x \ln x \, dx$ [2 marks]

- c) Solve the following equations using matrix algebra [6 marks]

$$2x + y + 3w = 15$$

$$x + 3y + w = 10$$

$$3x + 2y + 2w = 20$$

QUESTION FOUR (20 MARKS)

- a) Solve the following difference equations

i) $y_{t+1} = y_t + 4$ ($y_0 = 8$) [2 marks]

ii) $y_t = 3y_{t-1} + 4$ ($y_0 = 1$) [2 marks]

iii) $y_{t+1} = 0.5y_t + 12$ ($y_0 = 9$) [2 marks]

iv) $y_{t+1} = 3y_t + 4$ ($y_0 = 10$) [2 marks]

- b) For the general first difference equation given as follows:

$$y_{t+1} + \alpha y_t = \beta$$

i) Find the general solution in the case where ($\alpha \neq -1$) [6 marks]

ii) Decompose the general solution into two components, the complementary function y_c and the particular integral y_p and interpret each term. [4 marks]

iii) Which of the two components in (ii) above determines whether the equilibrium is dynamically stable or not [2 marks]

QUESTION FIVE (20 MARKS)

Consider a simple economy with three sectors. The inter-sector intermediate use of production is given by the input-output matrix A while final demand is given by matrix D.

$$A = \begin{bmatrix} 0.6 & 0.02 & 0.1 \\ 0.3 & 0.2 & 0.4 \\ 0.2 & 0.4 & 0.3 \end{bmatrix} \quad D = \begin{bmatrix} 250 \\ 150 \\ 100 \end{bmatrix}$$

- a) Determine gross outputs that will satisfy both the final demand and inter-industry needs (15 marks)
- b) Compute the comparative statics matrix that shows gross output needed for any changes in final demand (5 marks)