MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

SMA 361-THEORY OF ESTIMATION

DATE: 18/8/2021 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS

Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

(a) A random sample of five values is taken from a normal population with mean μ and variance δ^2both unknown. The values are: 1.07, 1.02, 1.04, 1.06, and 1.05. Estimate the population mean and population variance. (4 marks)

(b) $Let\ x_1, x_2, .. x_n$ be a random sample drawn from a population with mean μ (unknown).

Show that if $y = \sum_1^n a_i x_i$ is unbiased estimator for μ if $\sum_1^n a_i = 1$ (5 marks)

(c) Consider a normal distribution N(μ, δ^2). Let $\mu_1 = \bar{x}_n = \sum_1^n \frac{x_i}{n}$ and $\mu_2 = \bar{x}_m = \sum_1^m \frac{x_i}{m}$ where m>n, be estimators of μ. Determine which estimator is more efficient (6 marks)

(d) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf is $f(x, \theta) = \theta x^{\theta-1}$,0$< x < 1, \theta > 0$

Show that $t_1 = \prod_{i-1}^n x_i$ is sufficient for θ (4 marks)

(e) Let $x_1, x_2, .. x_n$ be a random sample of size n from a distribution with mean from a population with mean μ (unknown) and known variance δ^2.Determine the maximum likelihood estimator (MLE) of δ^2 (6 marks)

(f) Let $x_1, x_2, .. x_n$ be a random sample of size n from a distribution with mean from a population with mean μ and variance δ^2 , show that $\bar{x}$ is a consistent estimator of μ.

(5 marks)

QUESTION TWO (20 MARKS)

(a) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf

$$f(x) = \begin{bmatrix} (\theta+1)x^{\theta}, & 0 < \mathrm{x} < 1, \ \theta > 0 \\ 0 & elsewhere \end{bmatrix}$$

Obtain the moment estimator of θ (9 marks)

(b) A random sample of size n is drawn from a normal population with $(0,\delta^2)$. Determine the minimum variance unbiased estimator (MVUE) of δ^2 and its variance. (11 marks)

QUESTION THREE (20 MARKS)

(a) Use the following data to fit the regression line $y = \propto + \beta x$

x	-2	-1	0	1	2
y	0	0	1	1	3

(8 marks)

(b) If $x_1, x_2, .. x_n$ is a random sample of size n from a population whose pdf is given by

$$f(x,\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, ...$$

Determine the uniformly minimum variance unbiased estimator (UMVUE) of $e^{-\lambda}$ (12 marks)

QUESTION FOUR (20 MARKS)

(a) Let $x_1, x_2, .. x_n$ be a random variable of size n from a population which is normally distributed with mean μ and δ^2 both unknown. Determine the MLE of μ and δ^2 (8 marks)

(b) A random sample $(X_1, X_2, X_3, X_4, X_5)$ of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators to estimate μ

$$t_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$$

$$t_2 = \frac{X_1 + X_2}{2} + X_3$$

$$t_3 = \frac{2X_1 + X_2 + \lambda X_3}{3}$$

Where λ is such that t_3 is an unbiased estimator of μ

(i) Determine λ (4 marks)

(ii) Show that $t_1 and\ t_2$ are unbiased (4 marks)

(i) State giving reasons the estimators which is best among $t_1, t_2, and\ t_3$ (4 marks)

QUESTION FIVE (20 MARKS)

(a) A population has a known mean μ and a standard deviation δ of 2.45. If a sample of 900 has a mean of 3.15cm and the population is normal obtain 95% confidence intervals for the true mean. (10 marks)

(b) Let X be random variable that is binomially distributed with a pdf defined as

$f(x;\theta) = \binom{n}{x}\theta^x(1-\theta)^{n-x} \quad for\ x = 0,1 \ldots n, \theta \in (0,1)$.Determine the Crammer-Rao lower bound (10 marks)