



MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

MASTER OF SCIENCE IN MATHEMATICAL MODELING AND COMPUTATION

SMA 806: NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

DATE: 26/1/2022

TIME: 10.00-1.00 PM

INSTRUCTION

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Use Picard's method of successive approximation to find the value of y for the following

function $\frac{dy}{dx} = 2x - y, y(0) = 1.$ (5 marks)

- b) Solve the following BVP $y'' + 9y = \cos(x), y'(0) = 5, y(\frac{\pi}{2}) = -\frac{5}{3}.$ (5 marks)

- c) Use the Runge-Kutta method of order four and $h = 0.1$ to find the approximate solution of

$\frac{dy}{dx} = x^2 + y$ at $x = 0.1, 0.2$ and 0.4 . Given that $y = -1$ when $x = 0.$ (5 marks)

- d) Approximate the y value at $x = 0.4$ of the following differential equation

$\frac{dy}{dx} = 0.5y, y(0) = 1.0$ using the Adams-Moulton method. (5 marks)

- e) Locate and classify the singular point of the equation,

$(x^4 - 2x^3 + x^2)\frac{d^2y}{dx^2} + 2(x-1)\frac{dy}{dx} + x^2y = 0$ (10 marks)

QUESTION TWO (20 MARKS)

- a) Obtain the power series solution of the equation, $\frac{dy}{dx} = y$ in ascending powers of x . (10 marks)
- b) Solve the equation $\frac{dy}{dx} = y$ using the method of separation of variables. (7 marks)
- c) Verify that the two solutions are equivalent. (3 marks)

QUESTION THREE (20 MARKS)

- a) Show that $x = 0$ is an ordinary point for the equation $(x^2 - 1)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + xy = 0$ (4 marks)
- b) Find the power series solution for the given equation in (a). (16 marks)

QUESTION FOUR (20 MARKS)

- a) Use the second-order Taylors series method on $(2,3)$ for the initial value problem $\frac{dy}{dx} = -xy^2, y(2) = 1$. Take $h = 0.1$. compare the results obtained with the exact solution of $y = \frac{2}{x^2 - 2}$. (10 marks)
- b) Consider the following nonlinear two-point boundary-value problem $y'' + y = \frac{\cos(x)}{2 + 2y^2}, y(0) = 2, y(\frac{\pi}{2}) = 1$. Solve the equation by perturbation method. (10 marks)

QUESTION FIVE (20 MARKS)

- a) Use the modified Euler's method to solve the differential equation $\frac{dy}{dx} = x + y^2$ with $y(0) = 1$. Take the step size $h = 0.1$. (5 marks)
- b) Consider the function $\frac{dy}{dt} = -y$, with initial condition $y(t = 0) = 1$. Implement a Matlab program to solve the function using Euler's method. (15 marks)