MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SECOND SEXAMINATION FOR THE DEGREE IN BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING

SUPPLEMENTARY EXAMINATION

EEE 402: COMPLEX ANALYSIS FOR ELECTRICAL ENGINEERS

DATE: 29/8/2017 **TIME: 2:00 – 4:00 PM**

INSTRUCTION TO CANDIDATES:

Answer **Question One** and **Any Two** Other Questions.

QUESTION ONE COMPULSORY (30 MARKS)

a) Let $f(z)=\frac{z+1}{z^2+9}$

i) Determine the poles of $f(z)$ (2 marks)

ii) Calculate the residue s of $f(z)$ at its poles (2 marks)

b) Determine the Maclaurin's series expansion of $f(z)$ given by

$f(z)=e^z$ (3 marks)

c) Evaluate $\lim_{z\to 2}\frac{z^2-2z+4}{z-2}$ (3 marks)

d) Show that $z=0$ is a removable singularity of the function

$f(z)=\frac{1-\cos z}{z^2}$. (5 marks)

e) Prove that the function $f(z)=z^2$ is analytic (5 marks)

f) Determine the Laurent series for $f(z)=\frac{1}{z(z+5)}$ valid in the region $|z|<5$ (5 marks)

g) Show that $f(z)=\frac{1}{2}$ is differentiable at all $z\neq 0$ and find its derivative. (5 marks)

QUESTION TWO (20 MARKS)

a) Integrate $\frac{1}{z}$, around C, the unit circle $|z|=1$, taken in the counterclockwise direction.

(4 marks)

b) Given that $w=f(x)=\left(z^2+1\right)^{\frac{1}{2}}$. Show that $z=\pm i$ are branch points of $f(x)$

(6 marks)

c) Evaluate the integral $\int_C \frac{5z-2}{z(z-1)}dz$, where C is the positively oriented circle $\lfloor z\rfloor=2$

(10 marks)

QUESTION THREE (20 MARKS)

a) Find the residues of $f(z)=\frac{z+1}{z^2+9}$ at a singular points. (4 marks)

b) Calculate the residues for $f(z)=\frac{1}{z(z+1)}$. (4 marks)

c) Determine the Laurent's series for $f(z)=e^{\frac{1}{z}}$ valid in the region $0<|z|<\infty$ (5 marks)

d) Evaluate the integral $\int_C \frac{z^2-z+1}{z-1}dz$, where C (a). $|z|=\frac{1}{2}$, and $|z|=2$, oriented in the positive sense. (7 marks)

QUESTION FOUR (20 MARKS)

a) If m and n are integers, show that $\int_0^{2\pi} e^{im\theta}e^{-in\theta}d\theta=\begin{Bmatrix} 0 when m\neq n \\ 2\pi when m=n \end{Bmatrix}$ (5 marks)

b) Show that $z=0$ is a removable singularity of the function $f(z)=\frac{1-\cos z}{z^2}$. (5 marks)

c) Use the power series expansions for e^z and $\frac{1}{1+z}$, to obtain the power series representation of $\frac{e^z}{1+z}$, in the region $|z|<1$. (5 marks)

d) Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)}dz$, where C is a positively oriented circle $|z|=3$.

(5 marks)

QUESTION FIVE (20 MARKS)

a) Determine the Taylor's series for $f(z)=\frac{1}{z+z^4}$ valid for $|z|<1$. (3 marks)

b) Expand $f(z)=\frac{3}{z^2(z-3)^2}$ in a Laurent series at $z=3$ (7 marks)

c) Find the Cauchy principal value of $\int_{-\infty}^{\infty} \frac{x^2 + x}{(x^2+1)(x^2+4)} dx$ (10 marks)