



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 335: ODE I

DATE: 26/3/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS TO CANDIDATES

Attempt question *one* (*compulsory*) and any other *two questions*.

QUESTION ONE (30 MARKS) (COMPULSORY)

- a) Consider the differential equation $\sqrt{3 + \left(\frac{dy}{dx}\right)^2} = 2x$
- State the order of the differential equation (2 marks)
 - State the degree of the differential equation (2 marks)
- b) Consider the differential equation $(x^3 + y^3)dx - 3xy^2 dy = 0$
- Show that it is homogeneous (3 marks)
 - Use the substitution $y = vx$ to transform the equation into an equation containing v and x only (2 marks)
- c) The scientific law states that the acceleration of a particle during upward motion under gravity is $-g$. write down this law as a differential equation (3 marks)
- d) Solve the differential equation $\frac{dy}{dx} = \frac{x - y + 3}{2x - 2y + 5}$, given that it is non-homogeneous. (4 marks)
- e) Solve the Bernoulli's equation $\frac{dy}{dx} + y = xy^3$ (4 marks)
- f) Determine the particular integral for the equation $\frac{d^2 y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{2x}$ (5 marks)

- g) Solve the linear equation $\frac{d^4 y}{dx^4} + 2\frac{d^3 y}{dx^3} + \frac{d^2 y}{dx^2} = \sin 3x$ (5 marks)

QUESTION TWO (20 MARKS)

- a) Solve the differential equation $\frac{d^3 y}{dx^3} - \frac{d^2 y}{dx^2} - \frac{dy}{dx} = e^{3x}$ (6 marks)
- b) Solve the differential equation $x\frac{d^2 y}{dx^2} - \frac{dy}{dx} + 4x^3 y = x^5$ (7 marks)
- c) Solve the differential equation $(D - 4)y = e^{3x} x^2$ (7 marks)

QUESTION THREE (20 MARKS)

- a) Show that $\frac{xy}{x^2 + y^2}$ is a homogeneous function in x and y only (5 marks)
- b) Solve the differential equation $(3x + 2y - 5)dx + (2x + 3y - 5)dy = 0$ (8 marks)
- c) Solve the equation $(D - 1)(D + 3)(D - 5)y = \cosh x$ (7 marks)

QUESTION FOUR (20 MARKS)

Consider the differential equation $(4x + 3y^2)dx + 2xdy = 0$

- a) Show that this equation is not exact (4 marks)
- b) Calculate an integrating factor of the form x^n where n is a positive integer (3 marks)
- c) Using the integrating factor in (b) above solve the differential equation (13 marks)

QUESTION FIVE (20 MARKS)

- a) Determine the differential equation whose solution is $y = A \cos 8x + B \sin 8x$ (4 marks)
- b) Solve the differential equation $\frac{dy}{dx} = \sin x$ (3 marks)
- c) Solve $\frac{dy}{dx} + y \cot x = \operatorname{cosec} x$ (5 marks)
- d) Solve $(2D^2 + 2D + 3)y = x^2$ (8 marks)