MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION (ARTS, SCIENCE)

SST 201/SMA 362: OPERATIONS RESEARCH I

DATE: 24/3/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS

1. *Answer* ***Question*** **one** *and any other* ***two*** *questions.*
2. *You must have the following items for this paper:*
   - *Scientific Calculator;*
   - *Graph paper.*

QUESTION ONE (30 MARKS)

a) i Outline **four** conditions which must be satisfied for a problem to be solved using linear programming technique. (4 marks)

ii Outline any **four** sources of constraints in an optimisation problem involving linear programming. (2 marks)

b) Explain each of the following types of variables as used in linear programming:

i. Slack variable;

ii. Surplus variable. (4 marks)

c) A factory produces four types of motor vehicle spare parts – A, B, C and D. The factory wishes to maximise contribution to revenue. The sale prices per component part are 12, 10, 18 and 24 thousand Kenya shillings respectively. The factory employs 200 computerised machines, 150 mechanical machines, and 100 manual machines which are operated 40 hours in a week. The time it takes to produce one of each type of component is as shown in the table below:

Component		A	B	C	D
Hours	Computerised	6	5	3	8
	Mechanical	8	6	2	4
	Manual	5	4	6	2

Formulate a linear programming model for this problem. (6 marks)

d) Given the linear programming model below:

Minimise : $z = 8x_1 + 12x_2 + 5x_3$

Subject to : $2x_1 + 3x_2 + 6x_3 \geq 20$

$6x_1 + 8x_2 + 5x_3 \geq 40$

$7x_1 + 2x_2 + 4x_3 \geq 50$

With : $x_1, \; x_2, \; x_3 \geq 0$

Determine its symmetrical dual program. (2 marks)

e) A soft drink production plant produces two quantities of soda: 300 ml and 500 ml. The sale prices per bottle are Ksh 30 for 300 ml and Ksh 45 for 500 ml. The firm wants to determine the hourly production plan which maximises contribution to its sales revenue. The production constraints are as shown below:

	Machine hours	Labour hours	Colouring agent
300 ml	1	2	1
500 ml	2	1	1
Maximum available	120	140	80

A customer has placed an order that a minimum of 20 bottles of 300 ml must be produced per hour in conformity with their consumption.

(i) Formulate a linear programming model for the problem. (3 marks)

(ii) Using the graphical method, determine the optimum production plan which maximises the hourly revenue for the bottling firm. (6 marks)

(iii) Interpret the optimum solution obtained in (ii) above. (3 marks)

QUESTION TWO (20 MARKS)

A lamp production plant produces two products: florescent tubes and energy saving bulbs. Tubes are sold at Ksh 60 per unit while bulbs are sold at Ksh 80 per unit. The inputs are labour hours, machine hours, glass material and coating material.

- Tubes require 3 labour hours per unit while bulbs require 5 labour hours per unit, but a maximum of 900 hours is available per day.
- Tubes require 5 machine hours per unit per day while bulbs require 4 machine hours per unit per day, but a maximum of 1200 hours is available per day.
- Tubes require 25 grams of glass material per unit of tube while bulbs require 18 grams of glass material per unit of bulb, but a maximum of 4500 grams is available per day.

- Tubes require 1 gram of coating material per tube while bulbs require 1 gram of coating material per unit of bulb, but a maximum of 200 grams is available per day.
- The Ministry of Energy has issued a directive that a minimum of 40 bulbs must be produced per day in conformity with an energy saving strategy.

The manager of the firm wants to determine the daily production plan which maximises contribution to revenue. The production constraints are as shown in the table below:

a) Formulate a linear programming model for the problem. (6 marks)

b) Using the graphical method, determine the optimum production plan which maximises revenue for the firm. (8 marks)

c) Interpret the optimum solution obtained in (b) above. (6 marks)

QUESTION THREE (20 MARKS)

A fruit farm which produces three types of fruit juices – mango, orange and pineapple, has its weekly production plan modelled as an LP program as shown below.

Maximise : $z = 80x_1 + 50x_2 + 100x_3$

Subject to : $2x_1 + 3x_2 + x_3 \leq 4000$ Machine hours

$x_1 + x_3 \leq 1500$ Labour hours

$2x_1 + 4x_3 \leq 2000$ Colouring agent

$x_2 \leq 500$ Comesa trade agreement

With : $x_1, \; x_2, \; x_3 \geq 0$ Non-negativity conditions

a) Using the Simplex method, determine the optimum weekly production plan for the farm. (16 marks)

b) Interpret the solution obtained in (a) above. (4 marks)

QUESTION FOUR (20 MARKS)

Below is a linear programming model for a given problem. Use it to answer the questions that follow.

Minimise : $z = 180x_1 + 120x_2 + 80x_3$

Subject to : $2x_1 + 2x_2 + x_3 \geq 80$

$3x_1 + x_2 + x_3 \geq 70$

With : $x_1, \; x_2, \; x_3 \geq 0$

a) Determine the symmetrical dual of the linear programming model above. (4 marks)

b) Using the Simplex method, determine the optimum solution of the dual program for the linear programming problem above. (10 marks)

c) Using the relationship between a primal program and its symmetrical dual program, extract the optimum solution of the primal program from the optimum solution of its dual program. (6 marks)

QUESTION FIVE (20 MARKS)

An oil distribution company is required to meet the demands 50, 70, 100, 60 in thousand litres at various destinations from supplies 100, 60, 120 in thousand litres from various sources. The transport cost per unit amount (thousand Kenya shillings) over the various routes are as given in the matrix below:

$$C = \begin{bmatrix} 5 & 1 & 3 & 4 \\ 2 & 4 & 2 & 5 \\ 1 & 3 & 2 & 4 \end{bmatrix}$$

The company wants to meet the demand at destinations by transporting the oil at the cheapest cost possible. Using the northwest corner (NWC) rule, do the following for the transportation problem:

a) Determine the optimal solution that minimizes the cost of transport; (17 marks)

b) Interpret the optimum solution obtained in (i) above. (3 marks)