



MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR FIRST SEMESTER EXAMINATION FOR MASTER OF
SCIENCE (MATHEMATICAL MODELLING AND COMPUTATION)

SMA 800: FUNCTIONAL ANALYSIS

DATE: 16/5/2019

TIME: 2:00 – 5:00 PM

INSTRUCTIONS TO CANDIDATES

(a) Answer ALL the questions in Section A and ANY TWO Questions in Section B

QUESTION ONE (20 MARKS)

- a) Prove that in the set $\mathbb{R}$ of all real numbers define d on $\mathbb{R} \times \mathbb{R}$ by
$$d(x, y) = |x - y| \quad \forall x, y \in \mathbb{R}, d \text{ is a metric space.} \quad (5 \text{ marks})$$
- b) Take $X = (0, 1]$ as a sequence of $\mathbb{R}$. Then X is not complete. (5 marks)
- c) Show that $\|x\| = (\sum_1^\infty |x_n|^p)^{\frac{1}{p}} \geq 0$ is a norm in l_p . (5 marks)
- d) Show that $Tx = 2x - 1$ is not a linear operator. (5 marks)

QUESTION TWO (20 MARKS)

- a) Prove that $Tx = \frac{1}{x}$ is not uniformly continuous on the interval $(0, 1)$. (5 marks)
- b) Show that $p(x) = |x|$ is a semi-norm on $\mathbb{C}$. (5 marks)
- c) Let $T: X \rightarrow Y$, be defined by $T(x) = 3x$, then T is linear. Prove. (5 marks)
- d) Find the fixed points of the mapping $T: \mathbb{R} \rightarrow \mathbb{R}$ defined by $Tx = 3x^2 - 4$. (5 marks)

QUESTION THREE (20 MARKS)

- a) Let T be a bounded linear operator, then
- i) $x_n \rightarrow x$ [where $x_n, x \in D(T)$] implies $T: x_n \rightarrow Tx$, (5 marks)
 - ii) The kernel of T is closed. (5 marks)
- b) Prove that if Y is a Banach space then $B(X, Y)$ is a Banach space. (5 marks)
- c) A compact subset M of a metric space is closed and bounded. (5 marks)

QUESTION FOUR (20 MARKS)

- a) Show that $\int_a^b x(t)dt, x \in C[a, b]$ is a linear and bounded function on $C[a, b]$. (5 marks)
- b) Prove that if $T: X \rightarrow X$, where X is a normed linear space is continuous at the origin, then T is uniformly continuous. (5 marks)
- c) Let E be an inner product space. Then for arbitrary $x, y \in E$
- $$\|x + y\|^2 + \|x - y\|^2 = 2(\|x\|^2 + \|y\|^2) \quad (5 \text{ marks})$$
- d) Prove that if X and Y are Banach spaces and $T: D(T) \subset X \rightarrow Y$ is a closed linear operator, where $D(T) \subset X$. Then $D(T)$ is closed in X , the operator T is bounded. (5 marks)