MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS & PROGRAMMING

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

BACHELOR OF EDUCATION

SMA 261: PROBABILITY AND STATISTICS II

DATE: 14/12/2021 **TIME: 2.00-4.00 PM**

INSTRUCTION:
Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) The joint probability density function for x and y is given by $f(x,y) = \frac{1}{21}(x+y)$

$x = 1, 2, 3.$

$y = 1, 2$

i. Show that $f(x,y)$ is a probability distribution (3 marks)

ii. Hence or otherwise compute $p(x \leq 2)$ (3 marks)

b) Given that X and Y are jointly absolutely continuous, with joint density

$$f_{X,Y}(x,y) = \begin{cases} kx^3y, & x \geq 0, \quad y \geq 0, \quad x+y \leq 1 \\ 0, & \text{Otherwise} \end{cases}$$

Determine the value of k (5 marks)

c) If the joint probability density function of X and Y is given by:

$$f(x, y) = 3y, 0 \leq y \leq 1, 0 < x < 1.\text{Determine } \mathbf{F}\left(\frac{1}{3}, \frac{1}{2}\right)$$ (6 marks)

d) The joint pdf of x and y is given by $f(x, y) = k(1 - x)(1 - y), for\ 0 < x, y < 1.$

Determine the value of k (4 marks)

e) Consider the bivariate table below

		y		
		-1	0	1
X	-1	0.04	0.01	0.20
	1	w	0.03	0.60

i. Determine the value of w (2 marks)

ii. Determine the marginal density of x (3 marks)

f) If x and y are independent random variables with a joint p.d.f defined by:

$$f(x, y) = \begin{cases} 4xy & 0 < \text{x, y} < 1, \\ 0 & \text{elsewhere} \end{cases}$$

Determine the E(x, y) (4 marks)

QUESTION TWO (20 MARKS)

a) Suppose that X and Y are discrete random variables with a p.d.f as shown in the bivariate table below:

		Y		
		0	3	4
X	5	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
	8	$\frac{3}{7}$	0	$\frac{1}{7}$

Calculate

i. $\Pr(Y = 4 | X = 5)$ (3 marks)

ii. the $E(X | Y = 3)$ (5 marks)

b) If x and y have joint probability density function $f(x,y) = \frac{8x^2}{7y^3}$ for $1 \leq x, y \leq 2$

i. Determine the marginal distribution of x. (4 marks)

ii. Determine the conditional density function of x given $y = y$ (6 marks)

iii. Determine whether x and y are independent. (2 marks)

QUESTION THREE (20 MARKS)

a) Given that the joint density $f_{X,Y}$ is given by

$$f_{X,Y}(x,y) = \begin{cases} Cye^{-xy}, & 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{Otherwise} \end{cases}$$

i. Determine the value of C so that $f_{X,Y}$ is a joint density function. (5 marks)

ii. Compute the marginal density of y (5 marks)

b) Suppose X is a continuous and Y is discrete random variables with a joint p.d.f given by:

$$f(x,y) = \frac{yx^{y-1}}{3}, 0 < x < 1, y \in 1,2,3.$$ Determine

i. $f_y(y)$ (5 marks)

ii. $E(X)$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Let X and Y be jointly absolutely continuous, with joint density function f given by

$$f(x,y) = \begin{cases} 4x^2y + 2y^5, & 0 \leq x \leq 1,\ 0 \leq y \leq 1 \\ 0, & \text{Otherwise} \end{cases}$$

i. Show that it is a joint density function (3 marks)

ii. Compute $\Pr(0.5 \leq X \leq 0.7,\ 0.2 \leq Y \leq 0.9)$ (5 marks)

b) The joint pdf of x and y is given by that $f(x,y) = \begin{cases} 6x^2y, & 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$

Determine:

i. μ_x and μ_y (6 marks)

ii. d_x^2 and d_y^2 (6 marks)

QUESTION FIVE (20 MARKS)

a) If the continuous random variable X with a pdf defined by that

$$f_x(x) = \begin{cases} \frac{2}{9}(x+1), & -1 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the p.d.f of $Y = X^2$. (10 marks)

b) Let the probability density function of X_1 and X_2 be given by

$$f_{X_1,X_2}(x_1,x_2) = \begin{cases} e^{-(x_1+x_2)}, & \text{for } x_1 \geq 0, \quad x_2 \geq 0 \\ 0, & \text{Otherwise} \end{cases}$$

Consider two random variables Y_1 and Y_2 be defined in the following manner.

$$Y_1 = X_1 + X_2$$

$$Y_2 = \frac{X_1}{X \ + X_2}$$

Determine the joint density of Y_1 and Y_2. (10 marks)