MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE(MATHEMATICS)

BACHELOR OF EDUCATION

BACHELOR OF ARTS

SMA 336: ORDINARY DIFFERENTIAL EQUATION II

DATE: 15/12/2017 **TIME: 8.30-10.30 AM**

INSTRUCTIONS

Answer question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (30 MARKS)

a) Define the following terms

i. Boundary value problem (2 marks)

ii. General solution of an n^{th} order differential equation (2 marks)

iii. Particular solution of n^{th} order differential equation (2 marks)

b) i. Define the wronskian of asset of functions $F = \{f_1, f_2, \ldots f_n\}$ (2 marks)

ii. Show that the wronskian of the set of functions $\{\ 5, \cos^2 t, \sin 2t\ \}$ is 20 for all t (4 marks)

c) Solve the initial value problem $\dfrac{d^2y}{dx^2} + y = 0$, $y(0) = 3$, $y'(0) = -4$ (4 marks)

d) Consider the initial – value problem $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 3$. Determine whether the differential equation has a unique solution (4 marks)

e) Solve the system of differential equations below using the elimination method

$$\frac{dy}{dx} = x + 2y + 1$$
$$\frac{dy}{dx} = 3x + 2y + t$$
(5 marks)

f) Prove the Bessel function identity $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$ (5 marks)

QUESTION TWO (20 MARKS)

a) Show that the Legendre equation $(1 - x^2)y'' - 2xy' + p(p+1)y = 0$ where p is a constant, has a regular singularity as $|x| \to \infty$ (8 marks)

b) Use the method of Frobenius to solve the differential equation

$$2x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 3)y = 0$$ (12 marks)

QUESTION THREE (20 MARKS)

Consider the system of differential equation

$$\frac{dx}{dt} = -2x + 2x^2$$
$$\frac{dy}{dt} = -3x + y + 3x^2$$

a) Determine the critical points (4 marks)

b) State the nature of the critical point (4 marks)

c) Linearize the system (6 marks)

d) Solve the system for x *and* y (6 marks)

QUESTION FOUR (20 MARKS)

a) Use the Netani's method to solve the differential equation

$$3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz = 0$$ (7 marks)

b) Solve the system

$$\frac{dx}{dt} = 2x + y + 3e^{2t}$$
$$\frac{dy}{dt} = -4x + 2y + t\,e^{2t}$$

(13 marks)

QUESTION FIVE (20 MARKS)

a) Determine the critical point of the system

$$\frac{dy}{dx} = x - y^2$$
$$\frac{dy}{dx} = x^2 - y$$

(6 marks)

b) Consider the differential equation

$$\frac{d^3z}{dt^3} - 3\frac{d^2z}{dt^2} + 2\frac{dz}{dt} - z = \sin t \;\; ; \;\; z(0) = 1, z'(0) = 0 \;,\; z''(0) = 3$$

Reduce the equation to a system of first order differential equation (5 marks)

c) Use the matrix method to solve the system

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -4x + 4y$$

(9 marks)