MACHAKOS UNIVERSITY

University Supplementary Examinations

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR BACHELOR OF EDUCATION, BACHELOR OF SCIENCE AND BACHELOR OF ARTS

SMA 202: LINEAR ALGEBRA I

DATE:………………………………………………………….. TIME:……………………

INSTRUCTIONS: Answer QUESTION ONE and any other TWO QUESTIONS

QUESTION ONE 30 Marks (Compulsory)

a.) Let A *and* B be $n \times n$ skew symmetric matrices, namely $A^T = -A$ and $B^T = -B$. Prove that;

i.) $A+B$ is skew symmetric (2marks)

ii.) AB is skew symmetric (3marks)

b.) Show that $(A-B)(A+B) = A^2 - B^2$ *iff* $AB = BA$ (3marks)

c.) Evaluate $\begin{vmatrix} 1 & 3 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & 3 \end{vmatrix}$ (4marks)

d.) Given the system

$$x + 2y = 6$$
$$4x + 3y = 3$$

Solve the system using crammers rule (4marks)

e.) Show that the vectors $u = (\sin\theta\ ,\ \cos\theta)$ and $v = (\cos\theta\ ,\ -\sin\theta)$ are orthogonal (4marks)

f.) Determine the equation of the plane through the points $A(1\ ,\ 1, 1)\ ,\ B(5\ ,\ 0\ ,\ 0)$ *and* $C(3\ ,\ 2\ ,1)$. (5marks)

g.) Consider the matrix $A = \begin{pmatrix} -4 & 1 & -6 \\ 1 & 2 & -5 \\ 6 & 3 & -4 \end{pmatrix}$. Reduce the matrix to echelon form. (5marks)

Question two (20marks)

a. Solve the following system of equations using the Crammer's rule (8marks)

$x + 2z = 6$

$-3x + 4y + 6z = 30$

$-x - 2y + 3z = 8$

b. Determine the inverse of the following matrix if possible. $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

(7marks)

c. Determine the equation of the plane perpendicular to the line $\mathbf{n} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ which passes through the point $A(1,2,1)$ (5marks)

Question Three (20marks)

a.) Show that the line $y = x + 1$ is not a subspace of $\Re$ i.e $W = \{(x, y) / y = x + 1\} = \{(x, x+1) / x \in \Re\}$ (5marks)

b.) Calculate the area of a triangle having vertices at $P(1 , 3, 2)$, $Q(2 , -1 , 1)$ *and* $R(-1 , 2 , 3)$ (5marks)

c.) Determine the value of h such that $(1 , 3, 3)$, $(-2 , -6 , h)$ *and* $(3 , 9 , -1)$ are Linearly independent (5marks)

d.) Calculate the work done in moving an object a long a vector $\vec{A} = 3i + 2j - 5k$ if the applied force is $\vec{F} = 2i - j - k$ (5marks)

Question four (20marks)

a) Solve the system of linear equations below by Gauss-Jordan elimination method. (7marks)

$2x + 3y - 6z = 11$

$x - 2y + 3z = 9$

$3x + y = 7$

b) Use Gram-Schmidt processes to find an orthogonal basis spanning the same subspace of R^n as the vectors $< 0,2,1,-1 >$, $< 0,-1,1,6 >$ and $< 0,2,2,3 >$ (7 marks)

c) Find the basis and dimension of the set spanned by the vectors $(1,-1,2)$, $(0,5,-8)$, $(3,2,-2)$, and $(8,2,0)$. (6 marks)

Question four (20marks)

a.) Solve the following system using Crammers method

$$2x - 5y + 2z = 7$$

$$x + 2y - 4z = 3 \qquad \text{(10 marks)}$$

$$3x - 4y - 6z = 5$$

b.) Consider the system

$$x_1 + 3x_2 + 2x_3 = 3$$

$$4x_2 + 2x_1 + 2x_3 = 8$$

$$x_1 + 2x_2 - x_3 = 10$$

Solve using matrix inversion method (10marks)