



MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS & COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

SMA 301: REAL ANALYSIS II

DATE: 12/4/2023

TIME: 8:30 – 10:30 AM

INSTRUCTIONS TO CANDIDATES

Answer QUESTION ONE and ANY TWO Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Define the following terms
- i) Absolute convergence
 - ii) Lower darbox sum
 - iii) Upper darbox
 - iv) Conditional convergence
 - v) Radius of convergence (5 marks)
- b) Determine the radius of convergence of the following series $\sum \frac{1}{n^2} x^n$ in $\mathbb{R}$. (5 marks)
- c) Prove that if (X, ρ) is a metric space and $f: (X, \rho) \rightarrow (\mathbb{R}, \theta)$ is continuous and if $a, b \in X$ where $f(a) > 0$ and $f(b) < 0$. Then there are neighbourhoods $N(a)$, $N(b)$ of a , b respectively such that $f(x) > 0 \forall x \in N(a)$ and $f(x) < 0 \forall x \in N(b)$. (5 marks)
- d) Let x_n and y_n be convergent sequence of elements of $\mathbb{C}$ with limits x and y respectively. Prove that $\lim_{n \rightarrow \infty} (x_n + y_n) = x + y$ (5 marks)
- e) Determine the sum of the series $\sum_{n=0}^{\infty} 2 \sin\left(\frac{\pi}{6}\right)^n$ (5 marks)

- f) Prove that all the monotonic functions on bounded intervals are functions of bounded variations. (5 marks)

QUESTION TWO 20 MARKS

- a) Test the convergence or divergence of the series $\sum_{n \in \mathbb{N}} \frac{1}{n^p}$ if
- i) $p > 1$ (4 marks)
 - ii) $p < 1$ (4 marks)
 - iii) $p = 1$ (4 marks)
- b) Prove that if $\sum z_n$ is absolutely convergent then $\sum z_n$ is convergent the converse is not true. (5 marks)
- c) Determine whether the following series diverges or converges
 $\sum_{n=1}^{\infty} \frac{2n}{5n+4}$ (3 marks)

QUESTION THREE (20 MARKS)

- a) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{R} \\ 0 & \text{if } x \in \mathbb{R} \end{cases}$. Does the right and the left sided limit exist? (6 marks)
- b) Let $f(x) = \begin{cases} x + 4, & \text{if } 0 < x < 1 \\ x + 5, & \text{if } x \geq 1 \end{cases}$. Is the function continuous? (6 marks)
- c) If V is monotonic increasing on $[a, b]$ that is $x, y \in [a, b]$ and $x < y$, then prove that $v(x) \leq v(y)$. (8 marks)

QUESTION FOUR (20 MARKS)

- a) Prove that if $f \in Bv[a, b]$ and $D = v - f$ then D is monotonic increasing on $[a, b]$. (7 marks)
- b) Prove that if $f: [a, b] \rightarrow \mathbb{R}$ be bounded and α increasing on $[a, b]$ if p_1 and p_2 are any 2 partitions of $[a, b]$ then $l(f, \alpha, p_1) \leq U(f, \alpha, p_2)$. (7 marks)
- c) Prove that if $f \in Bv[a, b]$ if and only if it can be expressed as the difference of two monotonic increasing functions on $[a, b]$. (6 marks)

QUESTION FIVE (20 MARKS)

- a) Let $f(x) = \begin{cases} 2x + 1, & \text{if } x \in (0, 1) \\ -4, & \text{if } x \in (1, 2) \end{cases}$
 And $\alpha(x) = \begin{cases} x + 3, & \text{if } x \in (0, 1) \\ 6 - x, & \text{if } x \in (1, 2) \end{cases}$ is $f \in \mathcal{R}(\alpha)$ on $(0, 2)$. (7 marks)
- b) Define f by $f(x) = 1 \forall x \in [a, b]$ show that $f \in \mathcal{R}[a, b]$ such that $\int_a^b f(x) dx = b - a$. (7 marks)
- c) Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$, determine whether the series converges or diverges using the integral test. (6 marks)