MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONIC ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 301: ENGINEERING MATHEMATICS X

DATE: 6/12/2021 **TIME: 11.00-1.00 PM**

INSTRUCTIONS

Answer question ONE (COMPULSORY) and Any other TWO questions

QUESTION ONE (30 MARKS)

a) Calculate differences through the sixth order to observe the effect of adjacent ‘‘errors’’ of size 1

k	0	1	2	3	4	5	6	7
y_k	0	0	0	1	1	0	0	0

(5 marks)

b) Let $f(x) = \ln(1 + x)$, $x_0 = 1$ and $x_1 = 1.1$ use Lagrange linear interpolation to calculate an approximate value of $f(1.04)$ and obtain a bound on the truncation error (5 marks)

c) Given the function

$$\int_{0.1}^{1.1} e^{-x^2} dx$$

with step size $h = 0.1$ approximate the solution using: -

i. Composite Trapezoidal Rule (5 marks)

ii. Composite Simpson’s Rule (5 marks)

d) Obtain the unique solution to $2x^3 + 3x^2 - 6 = 0$ on the interval $[1,1.2]$, by Newton’s – Raphson method generate the sequence $\{p_n\}_{n=0}^{\infty}$ by: -

$$p_n = p_{n-1} - \frac{2p_{n-1}^3 + 3p_{n-1}^2 - 6}{6p_{n-1}^2 + 6p_{n-1}} \quad ; n \geq 1$$

(round your answers correct to nine decimals) (10 marks)

QUESTION TWO (20 MARKS)

a) The function f is displayed in the table below rounded to five correct decimals. We know that $f(x)$ behaves like $\frac{1}{x}$ when $x \to 0$.

x	$f(x)$	x	$f(x)$
0.1	20.02502	0.6	3.48692
0.2	10.05013	0.7	3.03787
0.3	6.74211	0.8	2.70861
0.4	5.10105	0.9	2.45959
0.5	4.12706	1.0	2.26712

We want an approximation of $f(0.55)$. Either we use quadratic interpolation in the given table or we set up a new table for

$g(x) = xf(x)$, interpolate in that table and finally use the connection $f(x) = \frac{g(x)}{x}$. Choose the one giving the smallest error, calculate $f(0.55)$ and estimate the error by using the forward difference table based on the nodes 0.5, 0.6, 0.7 and 0.8 for the function $f(x)$ using the quadratic interpolation method (12 marks)

b) Using the table below for the function $f(x) = xe^x$

x	$f(x)$	x	$f(x)$
1.8	10.889365	2.1	17.148957
1.9	12.703199	2.2	19.855030
2.0	14.778112	2.3	22.940620

approximate $f''(2.0)$ using the second derivative formula. Hence or otherwise calculate the actual errors (8 marks)

QUESTION THREE (20 MARKS)

a) Using the modified Newton's – Raphson method, find the solutions accurate to within 10^{-9} to the problem $f(x) = e^x - x - 1$ for $[0,1]$ [HINT: $p_0 = 1$] (8 marks)

b) Given the integral

$$\int_{0.1}^{0.2} \frac{x^2}{\cos x} dx$$

i. Compute the integral using Taylor development with an error $< 10^{-6}$ (6 marks)

ii. If we use the trapezoidal formula instead which step length of the form $10^{-k}, 2.0\ X\ 10^{-k}$ or $5.0\ X\ 10^{-k}$ would be the largest giving the accuracy above? How many decimal places would be required in function values? (6 marks)

QUESTION FOUR (20 MARKS)

a) The error function is defined by the integral

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt$$

x	0.05	0.10	0.15	0.20
$erf(x)$	0.05637	0.11246	0.16800	0.22270

Approximate $erf(0.08)$ by using

i. Linear Lagrange interpolation polynomial

ii. Second Lagrange interpolation polynomial

From the given table of correctly rounded values. Hence or otherwise estimate the total error (6 marks)

b) Considering the initial value problem given by $y' = 2x + 3y$, $y(0) = 1$

i. Find x if the error in $y(x)$ obtained from the first four terms of the Taylor series is to be less than $5.0\ X10^{-5}$ after rounding off (5 marks)

ii. Determine the number of terms in the Taylor series required to obtain results correct to $5.0\ X10^{-6}$ for $x \le 0.4$ (4 marks)

iii. Use Taylor's series second order method to get $y(0.4)$ with step length $h = 0.1$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Considering the following Runge-Kutta method for the differential equation $y' = f(x, y)$

$$y_{n+1} = y_n + \frac{1}{6}(K_1 + 4K_2 + K_3)$$
$$K_1 = hf(x_n, y_n)$$
$$K_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{K_1}{2}\right)$$
$$K_3 = hf(x_n + h, y_n - K_1 + 2K_2)$$

i. Compute $y(0.4)$ when $y' = \frac{y+x}{y-x}$, $y(0) = 1$ and $h = 0.2$ round your answer to five decimals (8 marks)

ii. What is the result after one step of length h when $y' = -y$, $y(0) = 1$ (4 marks)

b) Use the classical Runge-Kutta formula of fourth order to find the numerical solution at $x = 0.8$ for $\frac{dy}{dx} = \sqrt{x + y}$, $y(0.4) = 0.41$ assume the step length $h = 0.2$ (8 marks)