



MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

DECEMBER SESSION EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 430: NUMERICAL ANALYSIS II

DATE: 24/5/2019

TIME:

INSTRUCTIONS:

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Solve the system of equations below using the inverse of the coefficients matrix method (5 marks)

$$x_1 + 2x_2 - x_3 = 2$$

$$3x_1 + 6x_2 + x_3 = 1$$

$$3x_1 + 3x_2 + 2x_3 = 3$$

- b) Determine the Eigen values and the corresponding Eigen vectors of the following system. (5 marks)

$$10x_1 + 2x_2 + x_3 = \lambda x_1$$

$$2x_1 + 10x_2 + x_3 = \lambda x_2$$

$$2x_1 + x_2 + 10x_3 = \lambda x_3$$

- c) Solve the system of equations below using Doolittle's method. (5 marks)

$$2x + 3y + z = 9$$

$$x + 2y + 3z = 6$$

$$3x + y + 2z = 8$$

d) Solve the system of equations

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

Using the Gauss- Elimination method with partial pivoting

(5 marks)

e) Consider the following matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

i) Show that the matrix satisfies its own characteristics equation

(5 marks)

ii) Determine A^{-1}

(5 marks)

QUESTION TWO (20 MARKS)

Consider the differential equation $\frac{dy}{dx} = 2e^x y$, $y(0) = 2$. Calculate $y(0.2)$ using Adams predictor corrector formula by calculating $y(0.1)$, $y(0.2)$ and $y(0.3)$ using the Euler's modified formula.

QUESTION THREE (20 MARKS)

Consider the approximate value of $I = \int_{-1}^1 e^{-x^2} \cos x dx$ obtain the value using.

a) Gauss-Legendre integration method for $n = 2$

(10 marks)

b) Radau integration method for $n = 2$

(10 marks)

QUESTION FOUR (20 MARKS)

a) Calculate the largest Eigen –value for the matrix

$$\begin{bmatrix} 10 & 4 & -1 \\ 4 & 2 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

Also evaluate the Eigen vector corresponding to the Largest Eigen -value

(10 marks)

b) Consider the system of equations

$$\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Where a is a real constant

i) Determine the values of a , for which the Jacobi and Gauss-Seidel methods converge.

(7 marks)

ii) For $a = 0.5$ calculate the value of ω which minimizes the spectral radius of the SOR iteration method.

(3 marks)

QUESTION FIVE (20 MARKS)

a) Consider the initial value problem (I .v. p) given by

$$y' = 2x + 3y \quad ; \quad y(0) = 1$$

Use Taylors series second order method to get $y(0.4)$ with step size length $h = 0.1$

(10 marks)

b) Let $f(x) = \ln x$ and $x_0 = 1.8$ for $h > 0$, evaluate $f'(x)$

(5 marks)

c) Calculate $\int_0^{\frac{1}{2}} \frac{x}{\sin x} dx$ using Romberg integration with step size $h = \frac{1}{16}$

(5 marks)