



MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 107. ENGINEERING MATHEMATICS IV

DATE: 16/3/2022

TIME: 8.30-10.30 AM

INSTRUCTIONS:

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) Using the formal definition of a derivative, prove that $\lim_{x \rightarrow 2} 5x - 7 = 3$ (3 marks)
- b) Find the equation of the straight line parallel to the line $y = \frac{1}{3}x - 3$ and passing through the point $(-1, 3)$ (3 marks)
- c) Find the derivatives of the following functions
- i. $y = (2x + 7)^3$ (2 marks)
- ii. $f(x) = x^2 \cos 2x$ (3 marks)
- iii. $y = \frac{\sqrt{x+5}}{x^3}$ (3 marks)
- d) Find the derivative of the function $f(x) = \frac{1}{x+2}$ from the first principles (5 marks)
- e) Given the function $y = \frac{x^2 - x - 2}{x - 2}$, find
- i. Its domain (1 mark)
- ii. Range (1 mark)
- iii. The asymptotes (3 marks)
- f) Given the complex function $f(z) = z^3 + 3z - 7$, find $f'(z)$ (2 marks)
- g) Find the Taylor's series expansion about $x = 0$ for the function $f(x) = e^{2x}$ (4 marks)

QUESTION TWO (20 MARKS)

- a) Find the maximum and minimum of the function $f(x) = x^3 + x^2 - 2x$ and sketch it. (6 marks)
- b) The position of a particle at any given time t is described by the equation $S(t) = t^3 - 4t^2 + 5t$. Find an expression for the
- Velocity (3 marks)
 - Acceleration after 3 seconds (3 marks)
- c) Find $\frac{dy}{dx}$ when $x = 0$ for the equation $(x + 2)3^x$ (5 marks)
- d) Define what is meant by a continuous function (3 marks)

QUESTION THREE (20 MARKS)

- a) Evaluate the following limits
- $\lim_{x \rightarrow 4} \frac{x^2 - 5x + 4}{x - 4}$ (2 marks)
 - $\lim_{x \rightarrow \infty} \frac{5x^3 - 2x^2 + 4}{2x^3 - 3x + 10}$ (2 marks)
 - $\lim_{h \rightarrow 0} \frac{e^h - 1}{h}$ (2 marks)
 - $\lim_{x \rightarrow 0} \frac{\tan 5x}{4x}$ (3 marks)
- b) Evaluate the following antiderivatives
- $\int (5x - \frac{1}{x^2}) dx$ (3 marks)
 - $\int x(x^4 + 2)^2 dx$ (4 marks)
- c) Given that $x = \cos 2t$ and $y = \cos^2(2t)$, find $\frac{dy}{dx}$. (4 marks)

QUESTION FOUR (20 MARKS)

- a) A curve has a parametric equations $x = t^2$ and $y = 4t$. Find the equation of the normal at $(t^2, 4t)$ (5 marks)
- b) Find the equation of the tangent line to the curve $x^2 + y^2 = 1$ at the point $(\frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{2})$ (5 marks).
- c) Use Newton-Raphson method to find the real root near $x = 3$ of the equation $x^3 - 3x - 5 = 0$ correct to 3 decimal places. (5 marks)

- d) Evaluate $\int_{z=0}^{z=4+2i} \bar{z} \, dz$ from $z = 0$ to $z = 4 + 2i$ along the line joining $z = 0$ to $z = 2i$ and the line joining $z = 2i$ to $z = 4 + 2i$. (5 marks)

QUESTION FIVE (20 MARKS)

- a) Using logarithmic differentiation find the derivative of
- $y = \frac{x+2}{x^2+2x-3}$ (5 marks)
 - $e^{xy} = x^2$ (5 marks)
- b) Find the area under the curve bounded by $y = x^2 + 2x + 3$, the lines $x = 1$, $x = 4$ and $y = 6$ (6 marks)
- c) Find the slope of the curve $x^2y + xy^2 = 6$ at the point $(1,2)$ (4 marks)