MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR SCIENCE IN MATHEMATISC & COMPUTER SCIENCE

BACHELOR SCIENCE IN STATISTICS & PROGRAMMING

BACHELOR SCIENCE IN ACTURIAL SCIENCE

BACHELOR SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION

SMA 260: PROBABILITY & STATISTICS 1

DATE: 14/12/2021 **TIME: 8:30 – 10:30 AM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS)

a) Differentiate between discrete and continuous random variable (4 marks)

b) A random variable x has the Poisson distribution with the moment-generating function given below

$$M_X(t) = e^{\lambda\left(e^t - 1\right)}$$

Find the mean of the random variable x. (4 marks)

c) The University staff age analysis by the Human Resource department has establish that the University staff members age is normally distributed where 31% of the staff is below 45 years and 8% are over 64 years. Determine the mean and the standard deviation of the University staff age distribution. (6 marks)

d) Given that X is a random variable with *pdf*

$$f(x)=\begin{cases}\dfrac{1}{2\sqrt{x}} & 1<x<4\\ 0 & elsewhere\end{cases}$$

Determine the quartile deviation of the random distribution (4 marks)

e) Given that $F(x)$ is the CDF of a poisson distribution with parameter λ and that $F(2)=2F(1)$. Determine the value of λ (6 marks)

f) Suppose that x is a continuous random variable with probability distribution

$$f_X(x)=\frac{x}{18}, \quad 0\le x\le 6$$

Determine the probability distribution of the random variable $Y=2X+10$. (6 marks)

QUESTION TWO (20 MARKS)

a) The average score for mathematics test in a form four class of 100 students was 55 marks with a standard deviation of 15 marks. If a student is randomly picked from the class, determine,

i. The probability that the student scored between 45 to 60 marks. (4 marks)

ii. The number of students who sat for the supplementary examination if the pass mark was 30 marks (4 marks)

b) If X is a continuous random variable with a pdf defined as

$$f(x)=\begin{cases}\dfrac{x}{6}+\dfrac{1}{12} & 0<x<3\\ 0 & \text{otherwise}\end{cases} \quad \text{and} \quad y=5x+3$$

Determine $g(y)$ and G(y) (7 marks)

c) In a class of 100 students, 15 are left handed. If 10 students are randomly selected for mathematics contest determine the probability that

i. None is left handed

ii. At most three will be left handed. (5 marks)

QUESTION THREE (20 MARKS)

a) A discrete random variable X has a probability function given by:

	0	1	2	3	4	5
	$\frac{2}{20}$	$\frac{3}{20}$	x	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

Determine the value of x and then compute the (1 mark)

i. Mean (2 marks)

ii. Variance of the random variable in the probability distribution. (3 marks)

b) An examination has 10 multiple-choice questions, with 4 choices for each question but only one is correct. To pass a student must answer at least 6 questions correct. If an ill prepared student decided to select the answers randomly determine the probability that the student passed the examination (4 marks)

c) A discrete random variable X has a Poisson probability distribution and its probability mass function (p.m.f.) is given by:

$$f(x)=\begin{cases}\dfrac{\lambda^x e^{-\lambda}}{x!} & \text{for } x=0,1,2,\cdots, \text{ and } \lambda>0\\ 0 & \text{elsewhere}\end{cases}$$

Show that

i. $E(X) = var(X) = \lambda$ (6 marks)

ii. $M_x(t)=\exp\left[\lambda\left(e^t-1\right)\right]$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Outline *three* characteristics of the binomial probability distribution (3 marks)

b) Miss Rachael has established that her supper takes between 80 to 120 minutes after placing an order in one of the hotels in Machakos town. Determine

i. The mean and median time of waiting

ii. The probability that he waits for more than 90 minutes

iii. His waiting time is within one standard deviation of the mean waiting time.(7 marks)

c) The maximum time to complete a task in a project is 2.5 days. Suppose that the completion time as a proportion of this maximum is a beta random variable with $\alpha = 2$ and $\beta = 3$. What is the probability that the task requires more than two days to complete? (6 marks)

d) A container has 100 key holders of which 10 are defective. If 5 of these key holders are randomly selected and delivered to a customer, determine the probability that the customer will get

i. None defective key holder (2 marks)

ii. One defective key holder. (2 marks)

QUESTION FIVE (20 MARKS)

A continuous random variable x is given by the function

$$f(x) = \begin{cases} \frac{3}{32}(6x - x^2 - 5), & \text{for } 1 \le x \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

a) Show that the function $f(x)$ is a probability distribution function (*p.d.f.*). (3 marks)

b) Determine each of the following measures about the random variable x in the probability distribution:

i. the mode of x

ii. the median of x

iii. the mean of x

iv. the variance of x (14 marks)

c) Determine the probability below from the distribution: (3 marks)

$P\,(2 \le x \le 4)$