MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF ARTS

SMA 436: METHODS OF APPLIED MATHEMATICS II

DATE: 3/5/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS TO CANDIDATES

Answer QUESTION **ONE** and any other **TWO** QUESTIONS

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Given than the $\delta_j^i = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$ Expand $\delta_j^i x_i x_j$. (3 marks)

b) If a_{ij} are constants and $\delta_{pq} = \frac{\partial x_p}{\partial x_q}$, calculate the partial derivative $\frac{\partial}{\partial x_k}\left(a_{ij} x_i x_j\right)$.(3 marks)

c) Define the following terms

  i. Umbral index (1 mark)

  ii. Covariant tensor of rank two (2 marks)

d) Find the resolvent kernel associated with $k(x,t) = e^{x-t}$. (5 marks)

e) Prove that a symmetric tensor of rank two has only $\frac{1}{2}n(n+1)$ different components in n-dimensional space. (4 marks)

f) Define the Christoffel symbols of first and second kind. (2 marks)

g) i) Define Fredholm integral equation of the second kind and give its general form (2 marks)

  ii) Solve the Fredholm equation $u(x) = 1 + \int_0^1 x u(t) dt$ using successive approximation method with initial condition $u_0(x) = 1$ (5 marks)

h) Define a weakly singular integral equation. (3 marks)

QUESTION TWO (20 MARKS)

a) State the Quotient law. (2 marks)

b) Show that the expression $A(i,j,k)$ is a covariant tensor of rank three if $A(i,j,k)B^k$ is a covariant tensor rank two and B^k is a contravariant tensor (6 marks)

c) Write down the laws of transformation of the following tensors

i. A_{ij} (2 marks)

ii. C_{lm}^{ijk} (2 marks)

d) Show that $g_{ij}dx^i dx^j$ is an invariant (6 marks)

QUESTION THREE (20 MARKS)

a) Prove that the metric tensor g_{ij} is a covariant symmetry tensor of rank two (12 marks)

b) If $ds^2 = (dr)^2 + r^2(d\theta)^2 + r^2 sin^2\theta(d\phi)^2$ find the value of:

i) [22,1] and [13,3] (4 marks)

ii) $\left\{ \begin{matrix} 1 \\ 2 \quad 2 \end{matrix} \right\}$ and $\left\{ \begin{matrix} 3 \\ 1 \quad 3 \end{matrix} \right\}$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Given $x^i = (x^1, x^2, x^3)$ and $\bar{x}^i = (\bar{x}^1, \bar{x}^2, \bar{x}^3)$ are coordinates of a point in Cartesian and cylindrical coordinate systems respectively, find the metric, component of the first and second fundamental tensor in cylindrical Coordinates. (15 marks)

b) Use the method of successive approximation to solve the Fredholm equation

$u(x) = \sin x + \int_0^{2\pi} \sin x \cos u(t) dt$ with initial condition $u_0(x) = 1$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Solve the Voltera integral equation $\phi(x) = x - \int_0^x (x-t)\phi(t)dt$ with the initial condition $\phi_0(x) = 0$ (10 marks)

b) Solve the boundary value problem $y'' + y = F(x), y(0) + y'(0) = 0;$ $y(\pi) + y'(\pi) = 0$ and find the Green's function for this problem. (10 marks)