MACHAKOS UNIVERSITY

University Examinations for 2020/2021

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS & COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

SMA 400: TOPOLOGY

DATE: 17/8/2021 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS: Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS) COMPULSORY

a) Give the definition of a topological space. (4 marks)

b) Let $\tau_1 = \{X, \emptyset, \{1\}, \{3,4\}, \{1,3,4\}, \{2,3,4,5\}\}$ be a topology on $X = \{1,2,3,4,5\}$ and let $A = \{1,2,3\} \subset X$. Find A', the derived set of A. (5 marks)

c) Let $X = \{1,2,3,4\}$. Let $S = \{\{1\}, \{2,3\}, \{3,4\}\}$ be a collection of subsets of X. Give the topology generated by the set S. (5 marks)

d) Let X be a non-empty set. Define the following topologies

i) Trivial topology

ii) Discrete topology (2 marks)

e) Define the neighborhood of a point $p \in X$ where X is a topological space. (2 marks)

f) Let $X = \{a, b, c, d, e\}$ and $\rho = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{c\}, \{abc\}, \{a,c\}, \{b,c\}\}$.

Let $E = \{a, b, c\}$.

Find:

i) The closure of E (4 marks)

ii) The interior of E (4 marks)

g) Find the topology generated in R by the class of all closed sub-intervals of the type $[a, a+1]$
. (4 marks)

QUESTION TWO (20 MARKS)

a) Find the topology generated by the sub-basis $\beta = \{\{1\}, \{1, 3, 4\}, \{2, 3\}, \{3\}\}$, where $X = \{1, 2, 3, 4\}$ (6 marks)

b) Given any collection of subsets of a nonvoid set X. Will this collection serve as a base for the topology? (6 marks)

c) If N_1 *and* N_2 are two neighborhoods of x, show that $N_1 \cap N_2$ is also a neighborhood of x. . (4 marks)

d) In a topological space X, a subset A *of* X is open if only if its complement is closed. (4 marks)

QUESTION THREE (20 MARKS)

a) Consider the topology $\tau = \{X, \emptyset, \{1\}, \{3,4\}, \{1,3,4\}, \{1,3,4,5\}\}$ on $X = \{1,2,3,4,5\}$ and the subset

$A = \{b, d, e\}$ of X. Compute showing working out:

i) $Int(A)$ (3 marks)

ii) $Ext(A)$ (3 marks)

iii) $B'dary(A)$ (4 marks)

b) Let (X, τ) be a topological space. Define a base and sub-base for the topology τ on X. (2 marks)

c) Let $X = \{1,2,3\}$ and $\tau = \{\{1,2\}, \{2,3\}, \emptyset, \{2\}, \{1,2,3\}\}$. Give the base and sub-base for the topology τ on X. (4 marks)

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d) Given a subset A of a topological space, define what it means for p to be a limit point of A. (2 marks)

e) Let X be a topological space and $A \subseteq X$. Give the definition of the closure of A. (2 marks)

QUESTION FOUR (20 MARKS)

a) Let (X, ∂) be a metric space and A $\subset X$. Then prove that if P is a limit point of A every neighborhood of P contains infinitely many points distinct from P. (8 marks)

b) Give the definition of a homeomorphism between two topological spaces X and Y. (4 marks)

c) Let (X, ρ) and (Y, ρ^*) be topological spaces and $f: X \to Y$ be a function if $p \in X$ and$\{p\} \in \rho$ Show that f is continuous. (8 marks)

QUESTION FIVE (20 MARKS)

a) Let $X = \{a, b, c, d, e\}$ and $J = \left\{\emptyset, x\{a\}\{b, c\}\{b, c, d\}\{a, b, c\}\{a, d\}\{d\}\{a, b, c, d\}\right\}$ let

$Y = \{a, b, c, d\}$,

i) find the induced topology on Y

ii) Define relative topology

iii) Find the closed set in relative topology (6 marks)

b) Define hereditary as used in topological space (2 marks)

c) $T_{0\ and}\, T_1$ spaces are hereditary prove (4 marks)

d) Let (X, ρ) be a topological space (X, ρ) is a T_1 space iff each singleton subset $\{x\}$ is closed in (X, ρ) (4 marks)

e) Consider the discrete topology τ on $X = \{a, b, c, d\}$. Find a sub-base for τ which does not contain any singleton sets. (4 marks)