



# MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 106: MATHS III

DATE: 15/5/2019

TIME: 8:30 – 10:30 AM

## INSTRUCTIONS:

Answer question one and any other two

### QUESTION ONE (30 MARKS)

- a) The position vectors of  $A$ ,  $B$ , and  $C$  are  $i - j + k$ ,  $2i + j - k$ , and  $3i - 2j + k$  respectively. Find the area of  $\triangle ABC$  (3 marks)
- b) A force  $15\text{units}$  acts in the direction of the vector  $i - 2j + 2k$  and passes through a point  $2i - 2j + 2k$ . Find the moment of the force about the point  $i + j + k$  (4 marks)
- c) If  $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ , write the characteristic equation, hence obtain the characteristic root of the matrix  $A$  (4 marks)
- d) If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ , show that  $A^2 - 4A - 5I = 0$ , where  $I$ ,  $0$  are the unit matrix and the null matrix of order three respectively. Use this result to find  $A^{-1}$  (4 marks)

- e) Given  $a = 2i + 3j - k$ ;  $b = i - j - 2k$ ;  $c = i + 2j + 2k$ . Show that  $a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$  hence find  $Iax(bxc)I$  (5 marks)
- f) The sum of the three numbers is 6. when we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers to three times the first number we get 12. Use determinants to find the numbers (5 marks)
- g) If  $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ ;  $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$  and  $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$ . Verify that:-
- $(AB)C = A(BC)$  (3 marks)
  - $A(B + C) = AB + AC$  (2 marks)

### QUESTION TWO (20 MARKS)

- a) Find the value of  $\rho$  for which the vectors  $4i + \rho j + 2k$ ;  $-i + 4j + 3k$  and  $8i + j - 3k$  are coplanar (3 marks)
- b) Find the area of a parallelogram having diagonals  $3i + j - 2k$  and  $i - 3j + 4k$  (4 marks)
- c) Solve the linear system by Gaussian elimination method:-  
 $x - y + 2z = 3$   
 $x + 2y + 3z = 5$   
 $3x - 4y - 5z = -13$  (5 marks)
- d) If  $AX = B$ , solve using matrix method given that:-  
 $A \equiv \begin{bmatrix} 1 & 2 & 3 \\ 3 & -1 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ ;  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ; and  $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  (8 marks)

### QUESTION THREE (20 MARKS)

- a) Find the magnitude vector of  $\sqrt{201}$  which is perpendicular to  $2i - j + k$  and  $i - 3j - 5k$  (4 marks)
- b) A force of magnitude 5 units acting parallel to  $2i - 2j + k$  displaces the point of application from  $(1, 2, 3)$  to  $(5, 3, 7)$ . Find the work done (4 marks)
- c) Determine the nature of solutions of the linear system  
 $5x + 3y + 7z = 4$   
 $3x + 26y + 2z = 9$  hence solve it  
 $7x + 2y + 10z = 5$  (6 marks)

- d) Use elementary transformation to find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \quad (6 \text{ marks})$$

#### QUESTION FOUR (20 MARKS)

- a) Find the unit vector perpendicular to both  $\vec{a} + \vec{b}$  and  $\vec{b} + \vec{c}$  where  $\vec{a} = i + j - k$ ;  
 $\vec{b} = -i + 2j + k$ ; and  $\vec{c} = i + 2j + k$  (4 marks)
- b) The position vectors of the points  $A, B, C$  and  $D$  are  $3i - 2j - k$ ;  $2i + 3j - 4k$ ,  
 $-i + j + 2k$ ;  $4i + 5j + \lambda k$  respectively. If the four points lie in a plane find the value of  $\lambda$  (4 marks)
- c) If  $A + B = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 4 & 6 \\ 1 & 0 & 2 \end{bmatrix}$  and  $A - B = \begin{bmatrix} 1 & 7 & 5 \\ 0 & 2 & 4 \\ 5 & 4 & 2 \end{bmatrix}$  find  $A^T$  and  $B^T$  (4 marks)
- d) Determine the value of  $a$  and  $b$  for which the system
- $$\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$$
- i) Has a unique solution (4 marks)
- ii) Has no solution (2 marks)
- iii) Has infinitely many solutions (2 marks)

#### QUESTION FIVE (20 MARKS)

- a) If  $a = -i + j + k$  and  $b = 7i + 2j + k$ , find the projection of  $a$  in the direction of  $b$  (3 marks)
- b) Diagonalize the matrix  $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$  (17 marks)