MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MECHANICAL AND MANUFACTURING ENGINEERING

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

EMM 419: MECHANICAL VIBRATIONS

DATE: 12/8/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS

Answer question ONE and any other two questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) What do you understand by "degrees of freedom of a vibrating system"? (1 mark)

b) Describe two cases where vibration is desirable**.** (2 marks)

c) The response of a sub-critically damped vibrating mass-spring system is given by;

$$y(t) = Ye^{-\zeta\omega_n t}\cos\left(\beta\omega_n t - \phi\right)$$

where Y and ϕ are constants to be determined.

i. Assuming that the initial displacement and velocity are x_0 and v_0 respectively, determine the response equation of the system. (3 marks)

ii. If x_0 = 1/1000 m, v_0 = 1/1000 m/s, c = 10 Ns/m, k = 100 and m = 1 kg, write a MATLAB code that can be used to determine the response of the system. Remember to label your axes and provide a title (3 marks)

iii. Sketch the expected response. (1 mark)

d) The tractor-trailer combination of Fig. Q1 b) moves at constant advance velocity in a straight line on perfectly plane and level ground

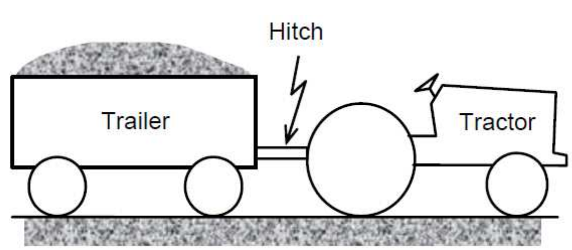


Fig. Q1 b)

Given that the mass of the trailer with its load is m_1, the mass of the tractor is m_2, the hitch is linearly elastic in the longitudinal direction with stiffness k, and all frictional forces are negligible:

i. Model the system as a two degree-of-freedom spring-mass system and write its equations of motion in matrix notation. (3 marks)

ii. Show that its non-trivial natural frequency is given by;

$$\omega_n = \sqrt{\frac{k}{m_{eq}}}$$

Where m_{eq} is equivalent mass (4 marks)

e) A vertical steel shaft 15 mm diameter is held in long bearings 1.5 metre apart carries at its middle a disc of mass 20 kg. If the modulus of elasticity for the shaft material is 200 GN/m^2, determine the critical speed of the shaft. (*For a shaft with fixed end carrying a concentrated load (W) at the centre, assume maximum deflection is given by* $\frac{Wl^3}{192\,EI}$) (3 marks)

f) In most cases, the analysis of torsional vibration assumes that the mass of the shaft is negligible. and therefore, the natural cyclic frequency is given by $f_n = \frac{1}{2\pi}\sqrt{\frac{K_e}{I}}$. Show that if the mass of the shaft is considered, the natural cyclic frequency of vibration will become;

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_e}{I+\frac{I_C}{3}}}$$ (4 marks)

Where;

I_c = Total mass moment of inertia of shaft

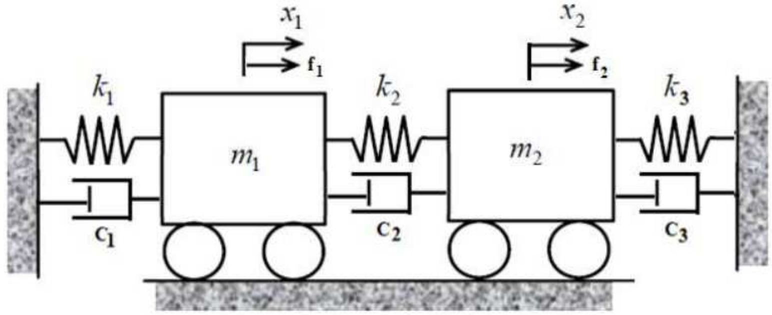
I = Mass moment of inertia of the rotor in kg.m^2

K_e = Torsional stiffness of the shaft in N-m

g) A cantilever shaft 60 mm diameter and 200 mm long has a disc of mass 100 kg at its free end. The Young's modulus for the shaft material is 200 GN/m^2. Determine the cyclic natural frequency of longitudinal and transverse vibrations of the shaft. (6 marks)

QUESTION TWO (20 MARKS)

Fig. Q2 shows a damped forced Two-DOF Vibration system. The system comprises of two masses m_1 and m_2, three springs k_1, k_2 and k_3 and viscous damping coefficients c_1, c_2 and c_3.



Fig, Q2

a) i Determine the equations of motion and write them in matrix notation (3 marks)

ii If the forcing functions and the damping effects are negligible, show that the natural frequencies of the system can be found from; (6 marks)

$$\omega_n{}^4 - \left[\frac{(k_1 + k_2)}{m_1} + \frac{(k_2 + k_3)}{m_2}\right]\omega_n{}^2 + \left[\frac{(k_1k_2 + k_2k_3 + k_3k_1)}{m_1m_2}\right] = 0$$

b) Taking $k_1 = 30$, $k_2 = 5$, $k_3 = 0$, $m_1 = 10$, $m_2 = 1$ and $c_1 = c_2 = c_3 = 0$ and the initial boundary conditions $x_1(0) = 1, \dot{x}_1(0) = x_2(0) = \dot{x}_2(0) = 0$, determine

i. the natural frequencies of the system (3 marks)

ii. the amplitude ratios (2 marks)

iii. free-vibration response of the masses m_1 and m_2 (6 marks)

QUESTION THREE (20 MARKS)

a) Fig. Q3 a), illustrates a simplified model used for studying vehicle ride characteristics. The front and rear suspensions are modelled to be purely linear elastic springs. The coordinates are chosen such that vehicle bounce is measured by the vertical motion of the vehicle body's centre of gravity.

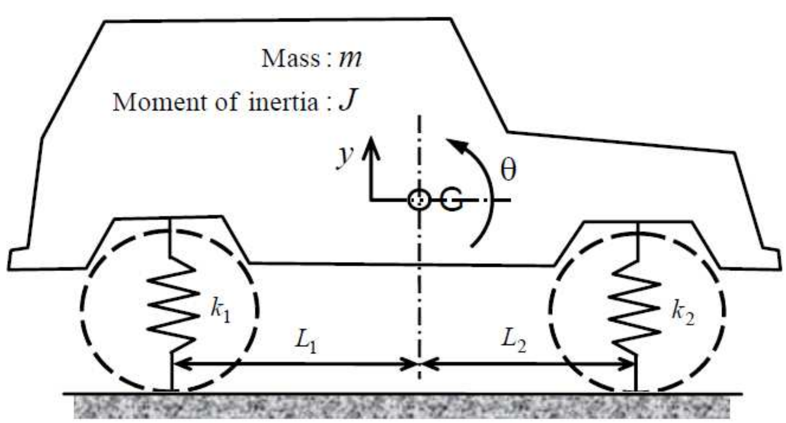


Fig. Q3 a)

Given that m = 1640 kg, k_1 = 30 kN/m, k_2 = 35 kN/m, L_1 = 1.35m, L_2 = 1.70 m and moment of inertia of the vehicle about the centre of gravity G = 2300 kgm^2, determine the natural circular frequencies of vibration of the system. (10 marks)

b) Fig. Q3 b) shows a seismic instrument

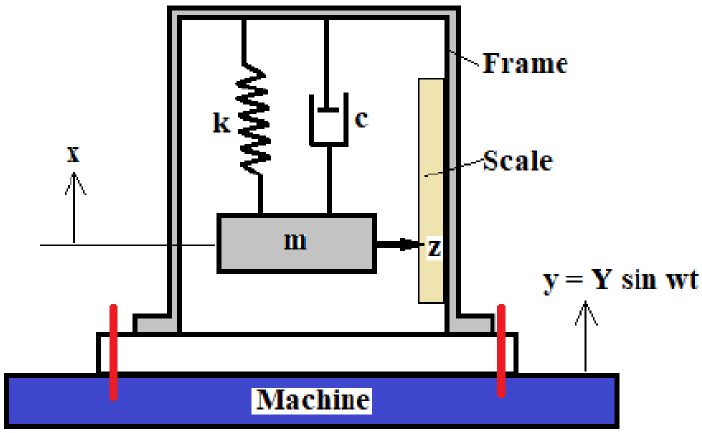


Fig. Q3 b)

i. What is a seismic instrument? (1 mark)

ii. Show that the relative displacement z of steady-state vibration is given by;

$$z(t) = \frac{Yr^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} sin(\omega t - \emptyset) \quad \text{(4 marks)}$$

c) An accelerometer has a suspended mass of 0.01 kg with a damped natural frequency of vibration of 140 Hz. When mounted on an engine undergoing an acceleration of 9.81 m/s^2 at an operating speed of 6050 rpm, the acceleration is recorded as 9.5 m/s^2 by the instrument. Find the damping coefficient of the accelerometer. (5 marks)

QUESTION FOUR (20 MARKS)

a) For a shaft of varying diameters d_1, d_2 and d_3 with lengths l_1, l_2 and l_3 respectively, Fig. Q4 a), show that such a shaft can be reduced to a torsionally equivalent shaft whose length is given by;

$$l = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4$$

(6 marks)

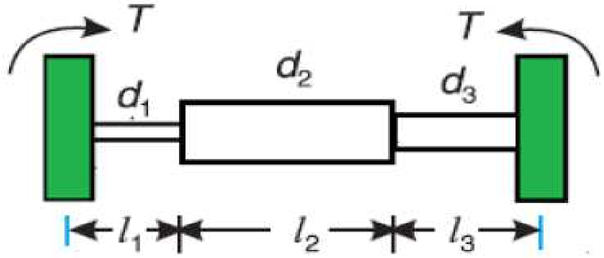


Fig. Q4 a)

b) A steel shaft ABCD 1.5 m long has two flywheels at its ends A and D. The mass of the flywheel A is 600 kg and has a radius of gyration of 0.65 m. The mass of the flywheel D is 800 kg and has a radius of gyration of 0.9 m. The connecting shaft has a diameter of 50 mm for the portion AB which is 0.4 m long and a diameter of 60 mm for the portion BC which is 0.5 m long and has a diameter of d mm for the portion CD which is 0.6 m long. Determine;

i. the diameter d of the portion CD so that the node of the torsional vibration of the system will be at the centre of the length BC (4 marks)

ii. the natural frequency of the torsional vibrations. Take G = 80 GN/m^2. (1 mark)

c) Study the following system showing torsional vibration of a geared system.

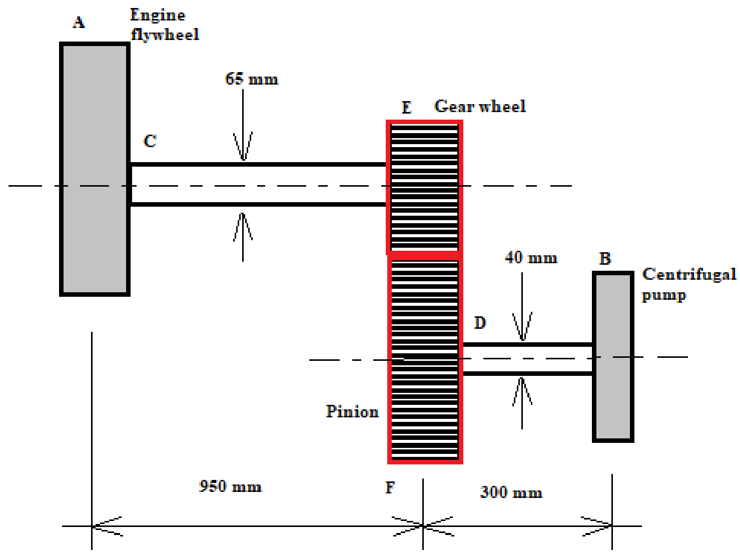


Fig. Q4 c)

The system is a reciprocating I.C. engine coupled to a centrifugal pump through a pair of gears. The shaft from the flywheel of the engine to the gear wheel is of 65 mm diameter and 950 mm length. The shaft from the pinion to the pump is of 40 mm diameter and 300 mm length. The engine speed is $^1/_4$th of the pump speed.

Moment of inertia of the flywheel – 810 kg/m^2

Moment of inertia of the gear wheel – 15 kg/m^2

Moment of inertia of the pinion – 4 kg/m^2

Moment of inertia of the pump – 17 kg/m^2

Modulus of rigidity for shaft material – 84 GN/m^2

Determine;

i. Position of the nodes (6 marks)

ii. the natural frequencies of torsional oscillation (3 marks)

QUESTION FIVE (20 MARKS)

a) A shaft 50 mm diameter and 3 metres long is simply supported at the ends and carries four loads of 1000 N, 1500 N, 750 N and a distributed load w of 200 N/m as shown in Fig. Q 5a). The Young's modulus for shaft material is 200 GN/m^2. Find the cyclic frequency of transverse vibration using Dunkley's method. (7 marks)

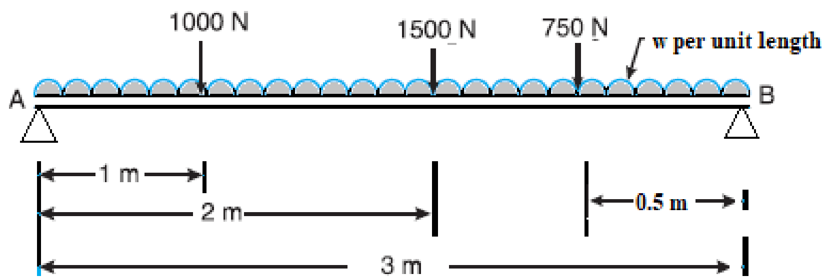


Fig. Q5 a)

b) A vertical shaft of 6 mm diameter is 300 mm long and is supported in long bearings at its ends. A disc of mass 50 kg is attached to the centre of the shaft. Neglecting any increase in stiffness due to the attachment of the disc to the shaft, find the critical speed of rotation and the maximum bending stress when the shaft is rotating at 75% of the critical speed. The centre of the disc is 0.25 mm from the geometric axis of the shaft. E = 200 GN/m^2. (6 marks)

c) Study the system shown in Fig. Q5 c) below of a free damped two degree of freedom system.

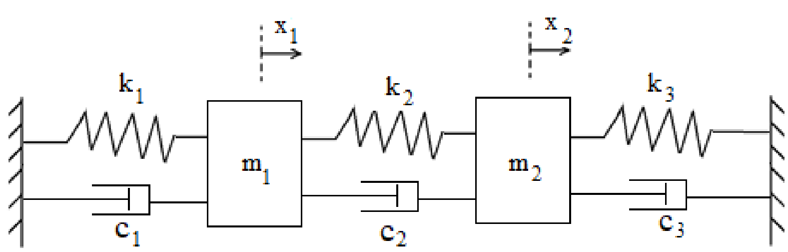


Fig. Q5 c)

Derive the state space formulations and hence, write a MATLAB code that can be used to plot the responses of displacements and velocities of the system if k_1 = 4000 N/m, k_2 = 2000 N/m, k_3 = 6000 N/m, c_1 = 1, c_2 = 2, c_3 = 0, m_1 = 1 kg, m_2 = 2 kg and initial displacement of m_1 = 0.01 m. (7 marks)