MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year
SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 361-THEORY OF ESTIMATION

DATE: 25/3/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS:

Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Explain the meaning of the following terms as applied in theory of estimation

i. Population

ii. An estimator (4 marks)

b) $Let\ x_1, x_2, .. x_n$be a random sample drawn from a population with mean μ (unknown).Show that if $y = \sum_1^n a_i x_i$ is unbiased estimator for μ if $\sum_1^n a_i = 1$ (6 marks)

c) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf is $P(X = x) = \frac{e^{-\mu}\mu^x}{x!}$, x=0,1,….Use the factorization theorem to show that $T = \sum x_i$ is a sufficient estimator of μ (4 marks)

d) $Let\ x_1, x_2, .. x_n$ be a random sample from a population with mean μ and variance δ^2.

Consider the following estimators forμ.

$$t_1 = {}^1/_2\,(x_1 + x_2)$$

$$t_2 = \frac{\frac{1}{2}x_1 + x_2 + \cdots + x_{n-1}}{2(n-1)}$$

$$t_3 = \bar{x}$$

- Show that each of 3 estimators is unbiased. (5 marks)
- Determine the efficiency of t_3 relative to t_2 (5 marks)

e) Let $x_1, x_2, .. x_n$ be a random sample of size n from a normally distributed population with mean μ (unknown) and known variance δ^2.Determine the maximum likelihood estimator (MLE) of μ . (6 marks)

QUESTION TWO (20 MARKS)

a) Use the following data to fit the regression line $y = \propto + \beta x$

x	-2	-1	0	1	2
y	0	0	1	1	3

(8 marks)

b) $Let\ x_1, x_2, .. x_n$ be a random sample of size n from a population whose pdf is given by $f(x, \theta) = \theta e^{-\theta x}$, x> 0,

Determine the Cramer-Rao lower bound for θ. (12 marks)

QUESTION THREE (20 MARKS)

a) Let $x_1, x_2, .. x_n$ be a random sample from $x \sim N(\mu_0, \delta^2)$ where μ, δ^2 are both unknown. Determine a joint sufficient statistic of $\mu, and\ \delta^2$ (8 marks)

b) A random sample of size n is drawn from a normal population with (0,δ^2). Determine the minimum variance unbiased estimator (MVUE) of δ^2 and its variance. (12 marks)

QUESTION FOUR (20 MARKS)

a) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf

$$f(x) = \begin{bmatrix} (\theta + 1)x^\theta, & 0 < \mathrm{x} < 1, \ \ \theta > 0 \\ 0 & elsewhere \end{bmatrix}$$

Obtain the moment estimator of θ (8 marks)

b) Given a random sample of size n from a population whose pdf is

$$f(x) = \frac{1}{2\sqrt{2\pi\theta}} e^{\frac{-1}{4\theta}(x-5)^2}$$

Obtain the maximum likelihood estimator (MLE) of θ and show that it's unbiased.

(12 marks)

QUESTION FIVE (20 MARKS)

a) A population has a known mean μ and a standard deviation δ of 2.45. If a sample of 900 has a mean of 3.15cm and the population is normal obtain a 98% confidence intervals for the true mean. (8 marks)

b) If $x_1, x_2, .. x_n$ is a random sample of size n from a population whose pdf is given by

$$f(x, \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots$$

Determine the uniformly minimum variance unbiased estimator (UMVUE) of $e^{-\lambda}$ (12 marks)