MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 104: CALCULUS

DATE: 22/1/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS TO CANDIDATES

Attempt question ***one (compulsory)*** and any other ***two questions***.

QUESTION 1(30 MARKS)

a) Consider the function $f(x) = \log_e(3x^2 - 4x + 5)$. Express the function as the composite of two functions (2 marks)

b) Consider the function

$$f(x) = \begin{cases} 7x - 2 & x \geq 2 \\ 3x + 5 & x < 2 \end{cases}$$

Determine the one-sided limits

i) $\lim_{x \to 2^+} f(x)$ (2 marks)

ii) $\lim_{x \to 2^-} f(x)$ (2 marks)

c) The function $f(x) : R \to R$ is defined by $f(x) = \dfrac{e^x - e^{-x}}{2}$. Calculate the inverse of $f(x)$

(3 marks)

d) Calculate $\lim_{x \to 4^-} \dfrac{3}{x - 4}$ (3 marks)

e) Prove that the function $f(x) = \sin^2 x$ is continuous for every value of x on R (4 marks)

f) A ladder $20m$ long leans against a vertical building. If the bottom of the Ladder slides away from the building horizontally at a rate of $2m/s$, how fast is the ladder sliding down the building when the top of the ladder is $12m$ above the ground. (4 marks)

g) Use the first principle to evaluate $\frac{dy}{dx}$ for $y = \log_b x$ (5 marks)

h) Calculate $\frac{d^2y}{dx^2}$ for the parametric equations $x = \cos 2t$, $y = 2\sin 2t$ (5 marks)

QUESTION TWO (20 MARKS)

Consider the function $f(x) = x^4 - 2x^2$

a) Determine using the second derivative

i.) Local maximum point (3 marks)

ii.) Local minimum point (3 marks)

b) Work out the coordinate of the points of inflection of $f(x)$ (4 marks)

c) Discuss the concavity of $f(x)$ (5 marks)

d) Sketch the graph of $f(x)$ (5 marks)

QUESTION THREE (20 MARKS)

Determine $\frac{dy}{dx}$

a) $5y^2 + \sin y = x^2$ (4 marks)

b) $y = \ln\left(\frac{x^2 \sin x}{\sqrt{1-x}}\right)$ (4 marks)

c) $ye^x = \sinh(xy)$ (4 marks)

d) $y = \tan^{-1}(2t+1),\ x = e^{-t^2}$ (4 marks)

e) $y = 3^{\cos x}(x^x)$ (4 marks)

QUESTION FOUR (20 MARKS)

a) A projectile is fired straight upwards with a velocity of $40 m/s$; its distance above the ground t sec after being fired is given by $S(t) = -16t^2 + 400t$. $S(t)$ is the distance of the particle from the ground after it has been fired.

i. Work out the acceleration at any time (t) (2 marks)

ii. Determine the maximum height achieved by the projectile (4 marks)

iii. Calculate the time and the velocity at which the projectile hits the ground (5 marks)

b) A vessel containing water is in the form of an inverted cone with semi-vertical angle of 30^0. there is a small hole at the vertex of the cone and the water is running out at a rate of $3cm^3$ per second. Calculate the rate at which the surface area in contact with water is changing when there are $81\pi cm^3$ of water remaining in the cone. (9 marks)

QUESTION FIVE (20 MARKS)

a) Determine;

i. $\lim\limits_{x \to 1} \dfrac{x^4 + 3x^3 - 13x^2 - 27x + 36}{x^2 + 3x - 4}$ (5 marks)

ii. $\lim\limits_{x \to -\infty} \dfrac{4x - 1}{\sqrt{x^2 + 2}}$ (5 marks)

iii. $\lim\limits_{x \to 0} \dfrac{|x|}{x}$ (5 marks)

b) Given that $f(x) = \tan^{-1} x$. Calculate $f'(x)$ (5 marks)