



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (COMPUTER SCIENCE)

SCO 109: LINEAR ALGEBRA FOR COMPUTER SCIENCE

DATE: 9/8/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS:

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE (30 MARKS) (COMPULSORY)

a) Find the cross product of the following vectors $(2, 4, -1)$ and $(3, 6, 1)$ (3 marks)

b) Reduce the following matrixes to be reduced echelon form

$$A = \begin{pmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 2 \\ 4 & -3 & 11 & 2 \end{pmatrix} \quad (3 \text{ marks})$$

c) Solve the following simultaneous equation using inverse matrix method

$$2x - y = 4$$

$$3x + 2y = 6 \quad (3 \text{ marks})$$

d) If $\begin{vmatrix} y & 5 \\ y & 2y + 1 \end{vmatrix} = 4$ find y (3 marks)

e) Calculate the cross product of the vectors $\vec{u} = (1, 2, 3)$ and $\vec{v} = (-1, 1, 2)$. (3 marks)

f) Find the inverse of the following matrix

$$\begin{bmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{bmatrix} \quad (4$$

marks)

- g) Show that the vector (11,3,-8) is a linear combination of the (1,1,0) and (2,1,-1)
- h) Calculate the x and y values for the vector (x, y, 1) that is orthogonal to the vectors (3, 2, 0) and (2, 1, -1). (4 marks)
- i) Determine if (1,2,3,1), (2,2,1,3) and (-1,2,7,3) is linearly dependent (4 marks)
- j) Find the angle between $u = 2i + 2j + 2k$ and $v = i + j + k$ (3 marks)

SECTION B

QUESTION TWO (20 MARKS)

- a) Show that $(A - B)(A + B) = A^2 - B^2$ iff $AB = BA$ i.e A & B commute (4 marks)
- b) Given $A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, show that $(AB)^T = B^T A^T$. (3 marks)
- c) Determine the values of x for which the determinant is zero where

$$\begin{pmatrix} x-2 & 4 & 3 \\ 1 & x+1 & -2 \\ 0 & 0 & x-4 \end{pmatrix} \quad (4 \text{ marks})$$

- d) Reduce $\begin{bmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 1 \\ 4 & -3 & 11 & 2 \end{bmatrix}$ to reduced row-echelon (canonical) form. (3 marks)

- e) Calculate the value of k for the vectors $\vec{u} = (1, k)$ and $\vec{v} = (-4, k)$ knowing that they are orthogonal. (3 marks)
- f) Show that the vectors $u=(1,-1,0)$ $v=(1,3,-1)$ and $w=(5,3,-2)$ are linearly dependent. (3 marks)

QUESTION THREE (20 MARKS)

- a) Let $a = 3i - j + k$ and $b = i + 2j - k$
 - i. Find $\vec{a} \times \vec{b}$
 - ii. Show that $\vec{a} \times \vec{b}$ is orthogonal to $\vec{a}$
 - iii. Show that $\vec{a} \times \vec{b}$ is orthogonal to $\vec{b}$ (6 marks)

b) Let $A = \begin{bmatrix} 2 & -5 & 2 \\ 1 & 2 & -4 \\ 3 & -4 & -6 \end{bmatrix}$

i. Find $\det A$

ii. Find $\text{Adj}(A)$

iii. Hence find A^{-1}

(6 marks)

c) Find the cross product of the following vectors $(2, 4, -1)$ and $(3, 6, 1)$

(4 marks)

d) Reduce the following matrixes to be reduced echelon form

$$A = \begin{pmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 2 \\ 4 & -3 & 11 & 2 \end{pmatrix}$$

(4 marks)

QUESTION FOUR (20 MARKS)

a) Solve using Cramer's rule.

$$x_1 + 3x_2 + 2x_3 = 3$$

$$2x_1 + 4x_2 + 2x_3 = 8$$

$$x_1 + 2x_2 - x_3 = 10$$

(5 marks)

b) Solve the following equations using inverse matrix method

$$x + 2y + z = 4$$

$$3x - 4y - 2z = 2$$

$$5x + 3y + 5z = -1$$

(5 marks)

c) Solve for x , y and z using Gauss-Jordan method

$$x + y + z = 5$$

$$x + 2y + 3z = 10$$

$$2x + y + z = 6$$

(5 marks)

d) Solve by elimination method

$$x_1 + x_2 + x_3 = 5 \dots$$

$$x_1 + 2x_2 + 3x_3 = 10 \dots$$

$$2x_1 + x_2 + x_3 = 0 \dots$$

(5 marks)

QUESTION FIVE (20 MARKS)

a) Define a vector space

(6 marks)

b) Show that $W = \{(x, y) / x = 2y\}$ is a subspace for $\mathbb{R}^2$.

(4 marks)

- c) Prove that the diagonals of a rhombus are perpendicular. (5 marks)
- d) Find the parametric and the symmetric equations of the line passing through the point $(2, 3, -4)$ and parallel to the vector $(3, 5, -6)$ (5 marks)