



MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (INFORMATION TECHNOLOGY)

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ECONOMICS

SMA 200: CALCULUS II

DATE: 18/8/2021

TIME: 8:30 -10:30 AM

INSTRUCTIONS

Attempt question *one (compulsory)* and any other *two questions*.

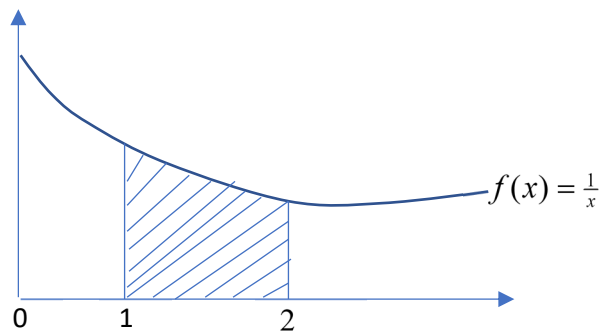
QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Consider the region bounded by the equations $y = \sin x$ and $y = 0$ from the line $x = 0$ to $x = \pi$. Calculate the centroid (center of mass) of the region (4 marks)
- b) A particle carrying a positive electrical charge $+q$ is released at a distance d from an immovable positive charge $+Q$. calculate amount of work done on the particle as its distance increases to $2d$. (4 marks)
- c) Consider the curves $y^2 = x$ and $y = x - 2$. Determine the area of the region bounded by these curves. (4 marks)
- d) Evaluate the improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ (4 marks)

- e) Determine
- $\int (x^2 + 2x)dx$ (4 marks)
 - $\int xe^x dx$ (5 marks)
 - $\int_0^{\frac{\pi}{2}} \cos^2 x dx$ (5 marks)

QUESTION TWO (20 MARKS)

- a) Consider the integral $\int \cos^n x dx$
- Prove that $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{(n-1)}{n} \int \cos^{n-2} x dx$ (4 marks)
 - Use the formula obtained in (i) above to evaluate $\int_0^{\frac{\pi}{2}} \cos^n x dx$; where n is a positive integer (6 marks)
- b) Consider the curve $y = \frac{1}{x}$ between $x = 1$ and $x = 2$ as in the figure below;



- Use the trapezoidal rule with 5 intervals to approximate the value of $\int_1^2 \frac{1}{x} dx$ (4 marks)
- Calculate the exact value of $\int_1^2 \frac{1}{x} dx$ by integration method (3 marks)
- Determine the percentage error generated by working out $\int_1^2 \frac{1}{x} dx$ using the Trapezoidal rule with the five intervals (3 marks)

QUESTION THREE (20 MARKS)

- a) Show that the volume of a hemisphere of radius r units is given by $V = \frac{2\pi r^3}{3}$ (4 marks)
- b) Prove that $\int \frac{dx}{x^2 + 2^2} = \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right)$ (4 marks)
- c) Work out the centroid of the region bounded by $y = x^2$ and $y = x$ (5 marks)
- d) Determine the total surface area of a sphere obtained by rotating $x^2 + y^2 = a^2$ between $x = -2$ and $x = 2$ (7 marks)

QUESTION FOUR (20 MARKS)

Evaluate;

- i) $\int e^{\sin x + x^3} (\cos x + 3x^2) dx$ (2 marks)
- ii) $\int \frac{dx}{\sqrt{9 - x^2}}$ (4 marks)
- iii) $\int \sec^2 x \tan x dx$ (4 marks)
- iv) $\int \frac{(x^2 + 5x + 6)}{(x + 2)} dx$ (5 marks)
- v) $\int_0^1 x^4 (\log x)^3 dx$ (5 marks)

QUESTION FIVE (20 MARKS)

- a) Evaluate;
- i) $\int_0^{\pi} \sin 10x \cos 8x dx$ (5 marks)
- ii) $\int \frac{3x + 7}{(x - 1)(x^2 + 1)} dx$ (6 marks)
- b) Compute $\int_0^1 \frac{\tan^{-1} x}{x^{\frac{3}{2}}} dx$ using Simpson's rule with 10 equal intervals. (9 marks)