



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION

SMA 200: CALCULUS II

DATE: 24/3/2021

TIME: 8.30-10.30 AM

INSTRUCTIONS

Attempt question *one* (*compulsory*) and any other *two questions*.

QUESTION ONE (30 MARKS)

- a) Consider the region bounded by the equations $y = \sin x$ and $y = 0$ from the line $x = 0$ to $x = \pi$. Calculate the centroid (center of mass) of the region (3 marks)
- b) A particle carrying a positive electrical charge $+q$ is released at a distance d from an immovable positive charge $+Q$. calculate amount of work done on the particle as its distance increases to $2d$. (3 marks)
- c) Determine the length of the curve $y = 3x^{3/2} - 1$ from $x = 0$ to $x = 1$ (4 marks)
- d) Calculate the area enclosed by the curves $x = y^2$ and $y = x - 2$. (4 marks)
- e) Evaluate the improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ (4 marks)
- f) Use fundamental theorem of calculus to evaluate $\frac{d}{dx} \left(\int_{1975}^{2x} \cos t \, dt \right)$ (4 marks)
- g) Determine

i. $\int x e^x dx$ (4 marks)

ii. $\int_0^{\frac{\pi}{4}} \tan^2 \theta d\theta$ (4 marks)

QUESTION TWO (20 MARKS)

a) Consider the integral $\int x^m (\log x)^n dx$

i.) Obtain its reduction formula (6 marks)

ii.) Use the formula obtained in (i) above to evaluate $\int x^4 (\log x)^3 dx$ (4 marks)

b) Obtain the reduction formulae for $I_n = \int \cos^n x dx$ (5 marks)

Hence show that;

$$\int_0^{\frac{\pi}{2}} \cos^n x dx = \begin{cases} \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{2}{3} & n \text{ odd} \\ \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{1}{2} \cdot \frac{\pi}{2} & n \text{ even} \end{cases} \quad (5 \text{ marks})$$

QUESTION FOUR (20 MARKS)

Evaluate;

a) $\int \cos 12x \sin 3x dx$ (3 marks)

b) $\int \sin^3 x dx$ (3 marks)

c) $\int x \sec^2 x dx$ (4 marks)

d) $\int \frac{dx}{\sqrt{9+x^2}}$ (4 marks)

e) $\int_0^2 \frac{10x-2x^2}{(x-1)^2(x+3)} dx$ (6 marks)

QUESTION FIVE (20 MARKS)

- a) Calculate the Simpson's approximation to $\int_0^1 \frac{2}{1+x^2} dx$ with $n = 10$. (10 marks)
- b) Determine the distance travelled in a straight line by a particle in 6 sec given that the velocity v in m/s is connected with the time as follows. Sketch the time velocity graph. (10 marks)

t	0	1	2	3	4	5	6
v	10	12	15	16	11	5	3