



# MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

DECEMBER SESSION EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

SMA 104: CALCULUS I

DATE: 27/9/2019

TIME: 2:00 – 4:00 PM

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## INSTRUCTIONS TO CANDIDATES

Answer Question one and any other two questions

### QUESTION ONE (COMPULSORY) (30 MARKS)

a) Evaluate the following limits

i).  $\lim_{x \rightarrow 1} (x^2 - 2x + 3)$  (2 marks)

ii).  $\lim_{x \rightarrow 0} \frac{(x^2 + 9)}{(x - 2)^2}$  (2 marks)

b) Find the  $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x}$  (3 marks)

c) Find the limit of  $\lim_{\theta \rightarrow \infty} \frac{\sin 2\theta}{\theta}$  (3 marks)

d) Find the derivative of  $f(x) = 2x^2 - x + 5$  from the first principal (3 marks)

e) Find the derivative of  $f(x) = \sin x$  using the 1<sup>st</sup> principal (4 marks)

f) Obtain the derivative of  $f(x) = 5x^2 - \cos x + 2$  (3 marks)

g) Show that  $\sin^2 \theta + \cos^2 \theta = 1$  (5 marks)

h) Differentiate  $\tan x$  using the first principle (5 marks)

## QUESTION TWO 20 MARKS

a) Evaluate

$$\lim_{x \rightarrow 0} x \sin \left( \frac{\sqrt{x+2}}{x} \right) \quad (3 \text{ marks})$$

$$\lim_{x \rightarrow 2} \sqrt{\frac{5x^3 - 15}{x^3}} \quad (2 \text{ marks})$$

b) For each of the following functions obtain their derivatives from first principal

i)  $f(x) = \frac{1}{2x+1}$  (3 marks)

ii)  $f(x) = 2x + 3$  (2 marks)

c) Obtain the derivative of the following functions

i)  $y = \sin^{-1} x$  (3 marks)

ii)  $y = \sin^3 x \cos^2 x$  (3 marks)

d) Find  $\frac{dy}{dx}$  for  $x^3 + 3x^2y + y^3 = 1$  (4 marks)

## QUESTION THREE 20 MARKS

a) Calculate the maxima and minima values of function  $y = x^3 - 3x^2 + 2$  and distinguish between them and sketch the graph. (5 marks)

b) A cylinder is to be constructed so that the sum of height and ball radius is 6cm. Denoting ball radius by  $r$ cm, volume  $v$ cm<sup>3</sup>. Show that  $r = \pi(6r^2 - r^3)$ . Hence show the value of  $r$  which make  $V$  a maxima. (5 marks)

c) Differentiate the following functions

i)  $y = \sin^{-1} x$

ii)  $\tan^{-1} x$  (10 marks)

#### QUESTION FOUR 20 MARKS

- a) Differentiate the following functions with respect to y
- i)  $(x + y)^3 = 2x - y$  (5 marks)
  - ii)  $x^3 + x^3y^4 = 4y$  (5 marks)
  - iii)  $x^3y + y^4\sqrt{x} + 4y = (x + y)^2$  (5 marks)
- b) Differentiate the following functions  $f(x) = (e^{2x})^{2x} + \cos 2x + \cos^{-1} x$  (5 marks)

#### QUESTION FIVE 20 MARKS

- a) find  $\frac{dy}{dx}$  for the following function
- i)  $x = \sin^3 \theta \quad y = \tan^2 \theta$  (5 marks)
  - ii)  $x = 3t^2 + 4t \quad y = e^{3t} + \cos^{-1} t$  (5 marks)
- b) Differentiate  $4xy + (4x^4y^3 + 12xy^5 = 12 + 12y)$  with respect to x (5 marks)
- c) Obtain the derivative of the following functions
- $f(x) = 3(2x + 2)^4 + x^3$  (5 marks)