MACHAKOS UNIVERSITY

SUPPLEMENTARY EXAMINATIONS 2022/2023

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONIC ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 300: ENGINEERING MATHEMATICS IX

DATE: **TIME:**

Instructions:

ANSWER QUESTION **ONE** (**COMPULSORY**) AND ANY OTHER **TWO** QUESTIONS.

Question one (30 marks)

a) Show that $\vec{a} \times \vec{b}$ is orthogonal to $\vec{a}$ and $\vec{b}$ (5 marks)

b) Prove that $Grad\,\vec{r} = \frac{\vec{r}}{r}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$ (5 marks)

c) If $\vec{V} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$, show that $\nabla \cdot (\nabla \times \vec{V}) = 0$ (4 marks)

d) Evaluate the scalar and vector projections of $\vec{b}$ onto $\vec{a}$ if $\vec{a} = (-2,3,-6)$ and $\vec{b} = (5,-1,4)$ (4 marks)

e) Find the directional derivative of $\varphi = (x + 2y + z)^2 - (x - y - z)^2$ at the point (2,1,-1) in the direction of $\boldsymbol{A} = \boldsymbol{i} - 4\boldsymbol{j} + 2\boldsymbol{k}$. (4 marks)

f) Verify Divergence theorem for $\vec{F} = x^2\boldsymbol{i} + y^2\boldsymbol{j} + z^2\boldsymbol{k}$ taken on the unit cube $0 \le x, y, z \le 1$ (5 marks)

g) Convert $\left(8, \frac{\pi}{4}, \frac{\pi}{6}\right)$ from spherical to Cartesian coordinates (3 marks)

Question Two (20 marks)

a) Show that $\lim_{t \to 1} \vec{r}(t) = \frac{t^2-t}{t-1}\hat{\imath} + \sqrt{t+8}\,\hat{\jmath} + \frac{Sin\,\pi t}{ln\,t}\hat{k}$ exists but $\vec{r}(t)$ is not continuous at $t = 1$ (5 marks)

b) Find $\frac{dz}{dt}$ if $z = ln(3x^2 + y^3)$, $x = e^{2t}$, $y = t^{\frac{1}{3}}$ (7 marks)

c) Find $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$ if $z = e^{xy}$, $x(u,v) = 3u\,sin\,v$, $y(u,v) = 4v^2u$ (8 marks)

Question Three (20 marks)

a) Find and classify the stationary points of $f(x,y) = x^2y + 3xy^2 - 3xy$ (10 marks)

b) Determine the volume of the solid that lies under the surface $z = xy$ and above the triangle with vertices (1,1), (4,1) and (1,2). (10 marks)

Question Four (20 marks)

a) Determine whether the vector field $\vec{F}(x,y) = (3x^2y - \cos x + 4, x^3 - e^y)$ is conservative and if so find its scalar potential function (10 marks)

b) Verify Green's theorem in the plane for $\oint_c (xy + y^2)dx + x^2\,dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$. (10 marks)

Question Five (20 marks)

a) Show that the scale factors in cylindrical coordinate system are given by $h_r = 1$, $h_\theta = r$ and $h_z = 1$. (6 marks)

b) Represent $\vec{A} = z\boldsymbol{i} - 2x\boldsymbol{j} + y\boldsymbol{k}$ in cylindrical coordinates and determine A_r, A_θ and A_z. (7 marks)

c) Evaluate $\iiint y dV$ over region E which is below the plane $z = x + 2$ and above the $xy-$plane and between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. (7 marks)