MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

………..YEAR …… SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN ……………………

SMA 404: COMPLEX ANALYSIS II

DATE: **TIME:**

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) State without proof the Residue theorem (2 marks)

b) Determine all the poles of the function $f(z)=\frac{z+2}{z^2\left(z^2+9\right)}$ stating the order of each pole.

(4 marks)

c) Evaluate the integral $\int_c \frac{5z-3}{z(z-3)}dz$, $|z|=3$ using the Residue Theorem (5 marks)

d) Find the transformation that maps the cycloid whose parametric equations are $=a(t-sint)$, $y=a(1-\cos t)$ into a straight line (6 marks)

e) Investigate the convergence of the product $\prod_{n=2}^{\infty}(1-\frac{1}{n^2})$ (6 marks)

f) Prove that $f_2(z)=\frac{1}{1+i}\sum_{n=0}^{\infty}\left(\frac{z+i}{1+i}\right)^n$ is an analytic continuation of $f_1(z)=\sum_{n=0}^{\infty}z^n$, showing graphically the regions of convergence of the series. (7 marks)

QUESTION TWO (20 MARKS)

a) State the Schwarz reflection principle (2 marks)

b) Determine whether the following functions satisfies the Schwarz- reflection principle

i) $f(z) = z^2 + 1$ (3 marks)

ii) $f(z) = z^2 + i$ (3 marks)

c) Prove that for closed polygons, the sum of the exponents $\frac{\alpha_1}{\pi} - 1, \frac{\alpha_2}{\pi} - 1, \ldots\ldots \frac{\alpha_{n-1}}{\pi} - 1$ in the Schwarz-Christaffel transformation is equal to -2. (5 marks)

d) Find the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2 (z^2 + 4)}$ at all its poles in the finite plane (7 marks)

QUESTION THREE (20 MARKS)

a) Differentiate between conformal mapping and isogonal mapping (3 marks)

b) Consider the function $u(x, y) = x^3 - 3xy^2$

i. Show that $u(x, y)$ is harmonic (2 marks)

ii. Determine the conjugate harmonic of $u(x, y)$ (5 marks)

c) Consider the lines $l_1 : x = 2$ and $l_2 : y = 2$ meeting at $z = z_o(2,2)$. Discuss the co formality of $T(z) = z^2$ at z_o. (5 marks)

d) Show that $f_1(z) = \sum_{n=0}^{\infty} z^n$ and $f_2(z) = \frac{1}{1-z}$ are analytic continuation of each other (5 marks)

QUESTION FOUR (20 MARKS)

a) Investigate the convergence of the product $\prod_{n=2}^{\infty}(1 - \frac{1}{n^2})$ (5 marks)

b) Show that $f_1(z) = \sum_{n=0}^{\infty} z^n$ and $f_2(z) = \frac{1}{1-z}$ are analytic continuation of each other (5 marks)

c) Consider the triangle A(0,1), B(1,1), C(1,0) on the z-plane with image under the transformation $T(z) = z^2$. Show that the transformation T(z) maintains the angle at A,B and C respectively. (10 marks)

QUESTION FIVE (20 MARKS)

a) Prove that the transformation of $u(x, y) = x^2 - y^2 + 2y$ is also harmonic under the transformation $z = w^3$ (10 marks)

b) Show that the mapping $f(z) = z^2$ is conformal at the point $z = 1 + i$ with $C_1: x = 1$ and $C_2: y = i$. Find the angle of rotation and sketch the curves. (10 marks)