MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING

EEE 402: COMPLEX ANALYSIS FOR ELECTRICAL ENGINEERS

DATE: 14TH JANUARY, 2025 **TIME: 8.30 AM – 10.30 AM**

INSTRUCTIONS:
Answer question ***ONE (Compulsory)*** *and* ***any other TWO*** *questions*

QUESTION ONE (30 MARKS)

a) Let $Z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$, express Z in polar form. (3marks)

b) Show that the $\lim_{Z \to 2i} \frac{Z^2 - 4}{Z - 2i} = 4i$ (3marks)

c) The function $f(z) = \begin{cases} Z^2, & if\ Z \neq i \\ 0, & if z = i \end{cases}$,examine continuity of the function at $z = i$. (4marks)

d) Show that $f(z) = 2\bar{z} + 5z$ is not analytic anywhere. (4marks)

e) Show that $u = x^2 - y^2 + 3x + 2$ is harmonic in a region R, and determine its harmonic conjugate. (8 marks)

f) Evaluate the contour integrals:

i) $\oint_C \frac{2z^2 + 1}{(z+i)^3} dz$ where C : $|z + i| = 3$ (4 marks)

ii) $\oint_C \frac{z^2 - z + 1}{(z-1)(z-4)(z+3)} dz$ where C : $|z| = 3$ (4 marks)

QUESTION TWO (20 MARKS)

a) Factorize the expression $2z^2 + 4z + 10$ (3 marks)

b) Solve the equation $z^5 - 1 = 0$ and exhibit the roots on the argand diagram. (6 marks)

c) Find the Laurent series of the function $f(z) = \frac{\sin z}{z-\pi}$ at z=π (4 marks)

d) Verify the Cauchy's integral Theorem for the function $f(z) = 3z^2 + iz - 4$ If C is the square with vertices at $1 \pm i, -1 \pm i$. (7 marks

QUESTION THREE (20 MARKS)

a) State without proof the Cauchy's residue theorem. (3marks)

b) Evaluate the following contour integrals:

i) $\oint_C \frac{e^{2z}}{z^2(z^2+2z+5)}dz$ where C : $|z| = 1$ (4marks)

ii) $\oint_C \frac{1}{z^2(z^2+2z+5)}dz$ where C : $|z+1-i| = 2$ (6marks)

iii) $\oint_C \frac{z^2}{(z^2+1)(z^2+4)}dz$ where C encloses the upper half-plane. (7marks)

QUESTION FOUR (20 MARKS)

a) Express $z = -5 + 5i$ in polar form. (4 marks)

b) Show that $\cos^{-1}(z) = -i\ln\left(z + \sqrt{z^2-1}\right)$. Hence find all solutions to the equation $\cos z = 2i$ (6 marks)

c) If $w = f(z) = \frac{2z+1}{3z-2}, z \neq \frac{2}{3}$, then find the values of w corresponding to $z = 2 - 2i$ (4 marks)

d) Find $\lim_{z \to -2i} \frac{(2z+3)(z-1)}{(z^2-2z+4)}$ (6 marks)

QUESTION FIVE (20 MARKS)

a) Find the fifth roots of the complex number $-4 + 4i$. (7 marks)

b) Define a simple closed path and make sketch of a simply connected domain. (3 marks)

c) Evaluate $\int_{1-i}^{2+i} (2x + iy + 1)dz$ along the curve $x = t + 1$, $y = 2t^2 - 1$ (6 marks)

d) Solve for $x \ and \ y$, Given that, $3x + 2iy - ix + 5y = 7 + 5i$ (4 marks)