MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SECOND SEMESTER EXAMINATIONS FOR BACHELOR OF EDUCATION

SMA 430: NUMERICAL ANALYSIS II

DATE: 11/12/2017 **TIME: 11.00-1.00 PM**

INSTRUCTIONS: Answer question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (30 MARKS)

a) Solve the system of equations below using the inverse of the coefficients matrix method

$$x_1 + 2x_2 - x_3 = 2$$
$$3x_1 + 6x_2 + x_3 = 1$$ (5 marks)
$$3x_1 + 3x_2 + 2x_3 = 3$$

b) Determine the Eigen values and the corresponding Eigen vectors of the following system.

$$10x_1 + 2x_2 + x_3 = \lambda x_1$$
$$2x_1 + 10x_2 + x_3 = \lambda x_2$$ (5 marks)
$$2x_1 + x_2 + 10x_3 = \lambda x_3$$

c) Solve the system of equations below using Doolittle's method.

$$2x + 3y + z = 9$$

d) $x + 2y + 3z = 6$ (5 marks)

$$3x + y + 2z = 8$$

e) Solve the system of equations

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

Using the Gauss- Elimination method with partial pivoting (5 marks)

f) Consider the following matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

i. Show that the matrix satisfies its own characteristics equation (5 marks)

ii. Determine A^{-1} (5 marks)

QUESTION TWO (20 MARKS)

a) Consider the differential equation $y'' + xy + y = 0$, $y(0) = 1$, $y'(0) = 0$. Calculate $y(0.1)$ using Runge-Kutta method. (10 marks)

b) Use Milne's predictor corrector method to obtain the solution of the equation $y' = x - y^2$ *at* $x = 0.8$ *given that* $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$ (10 marks)

QUESTION THREE (20 MARKS)

Consider the approximate value of $I = \int_0^1 \frac{dx}{1+x}$ using;

a) Simpson's rule with 8 equal sub-intervals (5 marks)

b) Trapezoidal rule with 8 equal sub-intervals (5 marks)

c) Lobatto method for $n = 2$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Consider the solution of the system of equations

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix}$$

Set up SOR scheme for the system (4 marks)

b) Determine the optimal relaxation factor and the rate of convergence (5 marks)

c) Perform three iterations of the SOR scheme given by

$$X^{(k+1)} = Hx^{(k)} + C, \quad k = 0,1,2,....$$ (5 marks)

d) Perform three iterations of the SOR scheme given by

$$(D + \omega L)V^{(k)} = \omega r^{(k)}$$

$$V^{(k)} = \omega (D + \omega L)^{-1} r^{(k)}$$

(6 marks)

Take the initial approximation as $x^0 = 0$. the exact solution is

$x_1 = 6$, $x_2 = 5$, $x_3 = 3$ (6 marks)

QUESTION FIVE (20 MARKS)

a) Determine the Least Square polynomial approximation of degree two for the function

$f(x) = x^{\frac{1}{2}}$ on $[0, 1]$ (8 marks)

b) Determine the Least Square straight line fit to the following data (12 marks)

x	$f(x)$
0.2	0.447
0.4	0.632
0.6	0.775
0.8	0.894
1.0	1.000