MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2023/2024

SCHOOL OF PURE AND APPLIED SCIENCES

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN APPLIED PHYSICS TECHNOLOGY, BACHELOR OF EDUCATION (SCIENCE)

SPH 400: CLASSICAL MECHANICS

DATE: TIME:

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section A is compulsory (30 marks).
- Answer any two questions from section B (each 20 marks)

SECTION A

Question 1 (Compulsory)

(a) Define the following terms
  (i) Torque (2 marks)
  (ii) Rigid body (2 marks)
  (iii) Curvilinear coordinate systems (2 marks)

(b) Distinguish between the following
  (i) Moment of inertia and product of inertia (2 marks)
  (ii) Discrete and continuous systems (2 marks)
  (iii) Degrees of freedom and constraints (2 marks)

(c) State the Hamilton's principle (2 marks)

(d) Determine the transformation from the cylindrical to rectangular coordinate system (4 marks)

(e) State the perpendicular axis theorem (2 marks)

(f) State the three laws of classical mechanics (6 marks)

(g) Two point masses, m_1 and m_2, connected by a rigid massless rod of length l are moving freely in a plane. Find an expression for the moment of inertia about
  (i) One of the masses (3 marks)

(ii) An axis perpendicular to the to the rod and going through the centre of mass
(3 marks)

SECTION B

Question 2

(a) (i) Give two properties of inertial frames of reference (2 marks)
(ii) State two examples of inertial frames of reference (2 marks)
(b) Show that a cylindrical coordinate system is orthogonal (6 marks)
(c) For a cylindrical coordinate system, using $\hat{e}_\rho$, $\hat{e}_\emptyset$ and $\hat{e}_z$, derive expressions for the
(i) Position vector, $\boldsymbol{r}$ (4 marks)
(ii) Velocity, $\boldsymbol{v}$ (3 marks)
(iii) Acceleration, $\boldsymbol{a}$ of the particle (3 marks)

Question 3

(a) Distinguish between (i) Conservative and non-conservative systems (2 marks)
(ii) Centre of mass and center of gravity (2 marks)
(b) Determine the number of degrees of freedom in each of the following cases
(i) Five particles moving on a plane (3 marks)
(ii) Two particles connected by a rod (3 marks)
(c) Determine the coordinates of the centre of mass of a system of particles with the following masses at the given points; 5g at (1,0,1), 3g at (0,3,2), 2g at (-1,0,3) and a 2g at (0,0,1).
(4 marks)
(d) For a system of particles whose internal forces are central forces, show that the total external torque on the system is equal to its time rate of change of angular momentum
(6 marks)

Question 4

(a) Distinguish between scleronomic and rheonomic systems (2 marks)
(b) (i) State the principle of virtual work (2 marks)
(ii) Verify the principle of virtual work (3 marks)
(c) Two masses m_1 and m_2, are connected by an inextensible string on a double incline that make angles α and β with the horizontal. Using the principle of virtual work, find the condition under which the system will be in equilibrium (5 marks)
(d) (i) State the D'Alembert's principle (2 marks)
(ii) Use the D'Alembert's principle, derive an expression for the acceleration of the of the masses in (c) above
(6 marks)

Question 5

(a) For a compound pendulum

(i) Set up its Lagrangian (5 marks)

(ii) Use the Lagrangian to obtain the equation to describe its motion (5 marks)

(b) A particle moves in the *xy* plane under the influence of a central force that depends only on its distance from the origin.

(i) Set up the Hamiltonian for the system (5 marks)

(ii) Write the Hamiltonian equations of the motion (5 marks)