Machakos University

University Examination for 2023/2024

SCHOOL OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

FIFTH YEAR EXAMINATIONS FOR BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONIC ENGINEERING

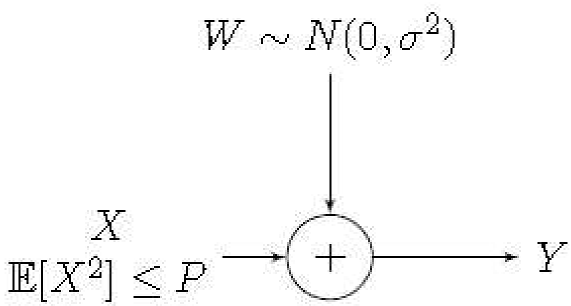
EEE 546: INFORMATION THEORY AND CODING

DATE: __ 2023_______ **TIME: 2 HRS**

<u>ANSWER QUESTION ONE AND ANY OTHER TWO</u>

QUESTION 1 (30MKS)

(a) Study the simple communication system below.



From this model, show that:

$$C(P) = \frac{1}{2} \log\left(1 + \frac{P}{\sigma^2}\right)$$

Where C(p) is the channel capacity per use and P is the signal power. (6mks)

(b) From observing a random variable *X* that takes its values from the discrete set $\Omega_X = \{a_1, a_2\ \ a_K\}$, write down an expression for the information obtained from a particular observation a_m. (2mks)

(c) Determine the entropy of the English text alphabet shown below. (4mks)

Source Symbol a_i	Probability p_i
Space	0.186
A	0.064
B	0.013
C	0.022
D	0.032
E	0.103
F	0.021
G	0.015
H	0.047
I	0.058
J	0.001
K	0.005
L	0.032
M	0.020
N	0.057
O	0.063
P	0.015
Q	0.001
R	0.048
S	0.051
T	0.080
U	0.023
V	0.008
W	0.017
X	0.001
Y	0.016
Z	0.001

(d) Given that (X1, X2) take on the values (0, 0), (1, 1), (1, 0). Compute the conditional entropy assuming that all the three data sets are equally likely. (5mks)

(e) Show that for the data sets below, the entropy is the same. (5mks)

$$U \in \{ \underbrace{1}_{\substack{\text{with} \\ \text{prob. } \frac{1}{2}}}, \underbrace{2}_{\substack{\text{with} \\ \text{prob. } \frac{1}{3}}}, \underbrace{3}_{\substack{\text{with} \\ \text{prob. } \frac{1}{6}}} \}$$

$$V \in \{ \underbrace{34}_{\substack{\text{with} \\ \text{prob. } \frac{1}{2}}}, \underbrace{512}_{\substack{\text{with} \\ \text{prob. } \frac{1}{3}}}, \underbrace{981}_{\substack{\text{with} \\ \text{prob. } \frac{1}{6}}} \}$$

(f) State the Shannon-Hartley Theorem with an explanation of its significance. Show that, according to this theorem, arbitrarily low bit-error rates cannot be achieved from a digital channel if E_b/N_0 is less than about -1.6 dB where E_b is the average energy per bit and $N_0/2$ is the two-sided power spectral density of additive white Gaussian noise. (8mks)

QUESTION 2 (20MKS)

(a) Given that:

$$X \sim U[a, b]$$

$$fX(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & otherwise \end{cases}$$

$$h(x) = \log(b - a)$$

Show that the differential entropy is given by:

$$\frac{1}{2}\log(2\pi\sigma^2 e)$$

For:

$$X \sim N(0, \sigma^2), f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-x^2}{2\sigma^2}}$$

(4mks)

(b) A particular medium transmits information using a bandwidth of 0.1 GHz. If the data rate desired is 0.5 Gbps;

(i) Does a signal to noise ratio of 1000 meet this requirement? (3mks)

(ii) Compute the signal to noise ratio needed for transmission at exactly 0.5 Gbps. (3mks)

(c) Given that X, Y, Z are three random variables that satisfy H(X, Y)=H(X) + H(Y) and H(Y, Z) = H(Z) + H(Y), is the following also true: H(X, Y, Z) = H(X)+H(Y)+H(Z). Give mathematical proof to back your answer. (4mks)

(d) Given that:

Y\X	0	1
0	1/2	1/4
1	1/8	1/8

i. Determine the joint entropy between the two variables X and Y.

ii. Determine the entropy for each variable.

iii. Compute H(X/Y) (6mks)

QUESTION 3 **(20MKS)**

(a) Given,

(x_1, x_2)	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$p(x_1, x_2)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

Determine the following: $(X_1|X_2 = 0)$, $(X_1|X_2 = 1)$, $(X_1|X_2)$, $(X_2|X_1)$. (8mks)

(b) Given a communication channel whose channel matrix is shown below, if the input source alphabet is X = {0, 1} and the two source bits have equal probability of being sent, and the output alphabet is Y = {0, 1};

$$\begin{pmatrix} 1-\epsilon & \epsilon \\ \epsilon & 1-\epsilon \end{pmatrix}$$

i. What is the entropy of the source, H(X)? (2mks)
ii. What is the probability distribution of the outputs, p(Y), and the entropy of this output distribution, H(Y)? (2mks)
iii. What is the joint probability distribution for the source and the output, p(X, Y), and what is the joint entropy, H(X, Y)? (2mks)
iv. What is the mutual information of this channel, I(X; Y)? (2mks)
v. How many values are there for ϵ for which the mutual information of this channel is maximal? What are those values, and what then is the capacity of such a channel in bits? (2mks)
vi. For what value of ϵ is the capacity of this channel minimal? What is the channel capacity in that case? (2mks)

QUESTION 4 (20MKS)

(a) Given two bit words $\mathbf{c}_1 = [1, 0, 0, 0]$ and $\mathbf{c}_2 = [0, 1, 0, 1]$. Determine the Hamming distance between the two codes. Assuming binary modulation with alphabet {+*a*,-*a*}, the corresponding symbol words or signal vectors are $\mathbf{s}1 = [+a,-a,-a,-a]$ and $\mathbf{s}2 = [-a, +a,-a, +a]$; determine the Euclidian distance based on this and derive an expression relating the Hamming distance with the Euclidian distance based on the binary modulation assumption. (6mks)

(b) Suppose data is to be transmitted at the rate of 0.2 Mbps through a certain transmission medium, whose bandwidth is 25KHz, what is the condition that must be met in this case as per the Nyquist capacity theorem. (5mks)

(c) Determine which of the following codes are linear. (3mks)

$$C_1 = \{00, 01, 10, 11\}$$
$$C_2 = \{000, 011, 101, 110\}$$
$$C_3 = \{00000, 11110, 10011, 01101\}$$
$$C_4 = \{101, 111, 011\}$$
$$C_5 = \{000, 001, 010, 011\}$$
$$C_6 = \{0000, 1001, 0110, 1110\}$$

(d) Determine the differential entropy given: (6mks)

$$f(x) = \tfrac{1}{2}\lambda \exp(-\lambda|x|)$$

QUESTION 5 **(20MKS)**

(a) Determine the differential entropy given: (5mks)

$$f(x) = \lambda \exp(-\lambda x), x \geq 0$$

(b) Consider a (6,3) linear block-code that is generated by a generator matrix:

$$
\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}
$$

Is the code systematic? Write down the check-bit equations and tabulate the code words and their weights. Find the parity check matrix **H** for this code. What is the minimum distance? How many errors can be:

i) corrected

ii) detected

with this code in hard decoding? How would you do the actual error correction detection?

(9mks)

(c) Given the message sequences m=00,01,10,11, determine the code sequences using the generator matrix G below. Determine the Hamming weight of the code sequences also.

(6mks)

$$
\mathbf{G} = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}
$$