MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 230: VECTOR ANALYSIS

DATE: 15/12/2017 **TIME: 2.00-4.00 PM**

INSTRUCTIONS

INSTRUCTIONS: Answer Question ONE and any other TWO.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) If two vectors $\vec{A} = 2\hat{i} + \sqrt{3}\hat{j} - 3\hat{k}$, $\vec{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ are inclined to each other at an angle θ. Find the value of θ. (4 marks)

b) Find the area of the triangle with vertices at $A(1,3,2)$, $B(2,-1,1)$, $C(-1,2,3)$. (5 marks)

c) Find the projection of the vector $7\hat{i} + 5\hat{j} + 2\hat{k}$ on the line passing though the points $A(2,3,-1)$, $B(-3,-5,4)$ (4 marks)

d) Given the position vector of any point on a curve is $\vec{r}(s) = x(s)\hat{i} + y(s)\hat{j} + z(s)\hat{k}$, show that $\frac{d\vec{r}}{ds} \bullet \frac{d^2\vec{r}}{ds^2} \times \frac{d^3\vec{r}}{ds^3} = \frac{\tau}{\rho^2}$ (5 marks)

e) Find the directional derivative of $\phi(x, y, z) = 4xz^3 - 3x^2y^2z$ at $(1,1,1)$ in the direction $3\hat{i} - 2\hat{j} + 5\hat{k}$ (4 marks)

f) Given the scalar function $\phi(x, y, z) = xy + y^2z$ and the curve C defined by $x = t^2$, $y = 2t$, $z = t + 5$ between the points $A(0,0,5)$, $B(1,2,6)$

i) Transform the integral $\oint_C \phi\, d\vec{r}$ to an integral involving t only (3 marks)

ii) Determine the interval of parameter t from the given parametric equations (2 marks)

iii) Lastly, evaluate $\oint_C \phi \, d\vec{r}$ along the given curve (3 marks)

QUESTION TWO (20 MARKS)

a) Prove that $\vec{A}\times(\vec{B}\times\vec{C})=\vec{B}(\vec{A}\bullet\vec{C})-\vec{C}(\vec{A}\bullet\vec{B})$ for $\vec{A}=(A_1,A_2,A_3)$, $\vec{B}=(B_1,B_2,B_3)$, $\vec{C}=(C_1,C_2,C_3)$ (7 marks)

b) Given $\vec{A}=3\hat{i}+5\hat{j}+2\hat{k}$, $\vec{B}=\hat{i}+3\hat{j}-2\hat{k}$, $\vec{C}=-\hat{i}+5\hat{j}+2\hat{k}$ illustrate that $(\vec{A}\times\vec{B})\times\vec{C}\neq\vec{A}\times(\vec{B}\times\vec{C})$. (5 marks)

c) Find a set of vectors reciprocal to the set $2\hat{i}+3\hat{j}-\hat{k}$, $\hat{i}-\hat{j}-2\hat{k}$, $-\hat{i}+2\hat{j}+2\hat{k}$ (8 marks)

QUESTION THREE (20 MARKS)

a) If $\vec{A}=e^{-2t}\hat{i}+\ln(3t^2+1)\hat{j}-\tan t\hat{k}$, find $\frac{d\vec{A}}{dt}$, $\frac{d^2\vec{A}}{dt^2}$ at $t=0$ and their magnitudes. (6 marks)

b) Prove the Frenet- Seret relation:

i. $\frac{d\hat{B}}{ds}=-\tau\hat{N}$ (5 marks)

ii. $\frac{d\hat{N}}{ds}=\tau\hat{B}-k\hat{T}$ (2 marks)

c) Find $\hat{T}$, k, $\hat{N}$ i.e. the unit tangent vector, the curvature and the unit normal respectively for the curve $\vec{r}(t)=-3\sin 2t\hat{i}-3\cos 2t\hat{j}$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Find an equation for the plane perpendicular to the vector $\vec{A}=6\hat{i}-3\hat{j}+2\hat{k}$ and passing through the terminal point $B(1,3,2)$ (4 marks)

b) Prove the following:

i. $\vec{\nabla}\bullet(\phi\vec{A})=(\vec{\nabla}\phi)\bullet\vec{A}+\phi(\vec{\nabla}\bullet\vec{A})$ for $\vec{A}=(A_1,A_2,A_3)$ and $\phi=\phi(x,y,z)$ (3 marks)

ii. $\vec{\nabla}\bullet(r^{-3}\vec{r})=0$ where $r=|\vec{r}|$ (3 marks)

c) Given that the curve $x = t$, $y = t^2$, $z = \frac{2t^3}{3}$, find the curvature and the binomial to the curve

(10 marks)

QUESTION FIVE (20 MARKS)

a) Determine the constants a, b, c such that

$\vec{V} = (x + cy + az)\hat{i} + (3x - y - bz)\hat{j} + (4x + 2y + 9z)\hat{k}$ is irrotational. (4 marks)

b) Evaluate $\iint_S \phi \hat{n} ds$ where $\phi = \frac{3}{8}xyz$ and S is the surface of the cylinder $x^2 + y^2 = 16$ between $z = 0$ and $z = 5$ in the first octant. (7 marks)

c) Verify the Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2 dy$, where C is the closed curve of the region bounded by $y = x$, $y = 2 - x^2$. (9 marks)