



MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF EDUCATION (SPECIAL NEED EDUCATION)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 330: NUMERICAL ANALYSIS I

DATE: 31/8/2022

TIME: 8.30-10.30 AM

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS)

- a) Show that the maximum absolute error in a number x is expressed as $\frac{E_{abs}}{|x^*|}$ (5 marks)
- b) Determine the second degree interpolating polynomial given that;
 $f(2) = 6, \quad f(4) = 8, \quad f(5) = 12$ (5 marks)
- c) Given the appropriate meanings of the numerical symbols for the forward finite difference Δ , the backward finite difference ∇ , and the central finite difference δ .
Show that at;
 $\delta = \nabla(1 - \nabla)^{-\frac{1}{2}}$ (5 marks)
- d) A real root of the equation $f(x) = x^2 - 5x + 1 = 0$ lies in the interval $(0, 1)$. Perform iterations of the Newton-Raphson method to determine this root. (5 marks)

- e) Consider the table below

x	0	2	4	6
$f(x)$	3	3	11	27

Determine $f''(1)$ (5 marks)

- f) Evaluate $I = \int_0^{\pi} t \sin t \, dt$ using trapezoidal rule with $n = 4$ (5 marks)

QUESTION TWO (20 MARKS)

Consider the data below:

y_i	1	3	8	11	31	69	113	223	351	521	739	1011
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Using the data above;

- Construct a forward finite difference table of order four (10 marks)
- Determine a misprint in the data (6 marks)
- Correct the misprint (4 marks)

QUESTION THREE (20 MARKS)

Consider the table below.

x	0	1	2	4
y_x	1	1	2	5

- Compute the divided differences (6 marks)
- Construct a divided difference table for the data (6 marks)
- Determine the collocation polynomial from the divided differences (8 marks)

QUESTION FOUR (20 MARKS)

Given that $y(0) = 1$, $y(1) = 0$, $y(2) = 1$ and $y(3) = 10$. Determine the;

- Finite difference table using the values (6 marks)
- Cubic interpolating polynomial for the values (7 marks)
- Value of $y''(4)$ using the values from the table in (a) above (7 marks)

QUESTION FIVE (20 MARKS)

- a) A particle moves along a straight line so that at time t seconds its distance s from a fixed point is given by;

$$\frac{ds}{dt} = t(\sqrt{8-t^3})$$

Using Simpson's Rule with 8 *strips* calculate the approximate distance travelled by the particle from $t = 0$ to $t = 2$ (8 marks)

- b) Consider the table below;

x	1.2	1.3	1.4	1.5	1.6
y	0.9320	0.9636	0.9855	0.9975	0.996

Determine;

- The value of x to 2dp for which y is maximum (7 marks)
- This maximum value of y (5 marks)